

# Aggregate Consequences of Dynamic Credit Relationships\*

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## Abstract

I investigate the aggregate consequences of canonical financial frictions in the supply of credit to firms: private information and limited enforcement. I propose a general equilibrium model in which entrepreneurs finance their firm through a long-term contract with a financial intermediary. The contract is inefficient because firm cash flow is costly to monitor and borrowers that repudiate cannot be excluded from capital markets. By investing in enforcement capacity, an intermediary can delay debt repayment and maintain incentive compatibility. Reforms that seek to decrease the cost of monitoring or enforcing contracts, or both, affect firm dynamics and can generate complementarities. Estimating the model with data on Colombian manufacturing firms in the 1980s and 1990s, I find that financial frictions are responsible for a significant aggregate output loss. Most of this distortion can be attributed to private information. Reforms that only reduce private information create significant economic growth and welfare gains, while those that only improve enforcement do not. There are significant complementarities between different types of reforms, as moral hazard is less significant when contracts are more enforceable.

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# 1 Introduction

Which financial frictions matter for aggregate resource allocation and for economic development and growth? A large microeconomic literature on firm and industry dynamics has studied financial frictions that arise when information about firms is expensive to acquire and when financial contracts are difficult to enforce. The cost of acquiring information and limits to contract enforceability are incentives for lenders to design different types of financial contracts [Levine, 2005]. These differences shape the supply of credit to firms and, in the aggregate, the efficiency at which the economy transforms savings into investment. Although there is broad consensus that financing frictions make it costly for firms to raise external finance (see Hubbard [1998] and Stein [2003] for surveys), there remains considerable debate surrounding the importance of financial frictions for the determination of macroeconomic outcomes.<sup>1</sup>

The main contribution of this paper is to propose and estimate a general equilibrium model in which different forms of financial frictions affect financial contracting and the supply of credit to firms. Recent developments in the theory on dynamic financial contracting have shown that private information and limited enforcement frictions can individually account for some of the empirical regularities on firm dynamics.<sup>2</sup> Although many properties of this class of financial contracts are well understood, their implications for the determination of macroeconomic outcomes remain largely unexplored.<sup>3</sup>

In the model, entrepreneurs with uncertain lifetimes can operate a long-lived firm. Starting a firm requires incurring a partially sunk fixed cost and working capital each

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<sup>1</sup> For example, Gilchrist, Sim, and Zakrajsek [2013] and Midrigan and Xu [2014] argue that the misallocation due to financial frictions is much lower than previous estimates.

<sup>2</sup> Empirical studies of firms have shown that smaller and younger firms pay fewer dividends, take on more debt, and experience more rapid but also more volatile growth, and that small and young firms' investment is more sensitive to cash flows (Cooley and Quadrini [2001], Cabral and Mata [2003], Oliveira and Fortunato [2006], Fagiolo and Luzzi [2006], and Lu and Wang [2010]). The seminal contributions on dynamic financial contracting to account for firm dynamics include Albuquerque and Hopenhayn [2004], Quadrini [2004], and Clementi and Hopenhayn [2006]. See also Hopenhayn and Werning [2008], Li [2013], Kovrijnykh [2013], and Popov [2014].

<sup>3</sup>The seminal work in this area is Cooley, Marimon, and Quadrini [2004] and Smith and Wang [2006]. More recent work includes Quintin [2008] on the size distribution of manufacturing establishments, Li [2010] on employment flows, Dyrda [2014] on the effect of aggregate uncertainty shocks, Gross and Verani [2011] and Dovis and Brooks [2014] on international trade, and Veracierto [2014] on consumers' decisions.

period. New entrepreneurs do not have enough resources to finance their firms and may obtain external financing by entering into a long-term contract with a financial intermediary. Once started, firms generate a random cash flow that is a function of the working capital input. Borrowing constraints emerge because firms operate under limited liability, firm cash flow is costly to monitor, and entrepreneurs repudiating their debt are not excluded from capital markets. A financial intermediary may invest in enforcement capacity to reduce the value of the entrepreneur's outside option, but doing so is costly.

Financial development in the model is a combination of lower costs of monitoring firms and investing in enforcement capacity. A less financially developed economy is subject to more severe moral hazard and limited enforcement frictions because the costs of monitoring firms and enforcing contracts are higher. Reforms to increase financial development in the model aim to reduce either one or both of these costs, which expands the set of feasible contracts and can thus have significant effects on firm dynamics and the distribution of firms.

The financial arrangement at the core of the general equilibrium model adds to the literature on dynamic contracting by considering the role of multi-period contracting, limited enforcement, and imperfect capital market exclusion in an environment with private information.<sup>4</sup> In the benchmark economy, investing in enforcement capacity and monitoring is not feasible because it is too costly, and the resulting optimal contract can be interpreted as the union of the optimal long-term contract that arises under private information studied by [Quadrini \[2004\]](#) and [Clementi and Hopenhayn \[2006\]](#) and the optimal long-term contract that arises under limited enforcement studied by [Albuquerque and Hopenhayn \[2004\]](#). Consistent with the predictions of these models, younger and smaller firms tend to grow disproportionately faster than older and larger firms, and the growth of smaller firms is also more volatile.

New contract structures arise from the interaction of private information and limited enforcement frictions when the cost of investing in monitoring and enforcement capacity

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<sup>4</sup>See previous work by [Atkeson \[1991\]](#); [Khan \[2001\]](#); [Castro, Clementi, and Macdonald \[2009\]](#); [Buera, Kaboski, and Shin \[2011\]](#); [Greenwood, Sanchez, and Wang \[2013\]](#); and [Arellano, Bai, and Zhang \[2012\]](#).

is sufficiently low. Investing in enforcement capacity under private information has two effects on the optimal contract. First, greater enforcement capacity provides greater insurance against debt repudiation by decreasing the value of the entrepreneur’s outside option. Second, a lower value of debt repudiation allows the intermediary to delay debt repayment while maintaining incentive compatibility at smaller firms, which increases the supply of credit to these firms. Moreover, when the cost of investing in monitoring is sufficiently low, financial intermediaries always monitor young and small firms while only relying on state-contingent promised continuation values to discipline moral hazard at older and larger firms. These mechanisms are central to the complementarities that arise in the aggregate between financial reforms that decrease the cost of monitoring and enforcing contracts.

I quantitatively investigate the aggregate effects of financial frictions by calibrating and estimating the parameters of the benchmark model economy using data from Colombia in the 1980s and 1990s and by conducting a series of counterfactual reform experiments. Colombia in this period provides an ideal benchmark to analyze the effect of different types of financial reforms, as a number of institutional and legal features prevented efficient firm monitoring and contract enforcement. An innovation in this paper is to use a simulated method of moment estimation (SMM) procedure more widely used in the dynamic corporate finance literature to structurally estimate the contract parameters using firm-level data.<sup>5</sup>

The quantitative analysis yields five results. First, financial frictions lead to a substantial misallocation of resources. Aggregate output in the benchmark model economy is about 15 percent lower than its potential first-best level. Although limited enforcement frictions can by themselves cause a 7 percent aggregate output loss, their relative effects in the presence of private information are small—that is, private information absent any enforcement frictions leads to roughly the same output loss as in the benchmark economy. Second, financial development can lead to significant economic growth. A reform that eliminates private information frictions leads to 10 consecutive years of economic growth at an average growth rate of about 1 percent per year before the

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<sup>5</sup>See [Strebulaev and Whited \[2012\]](#) for a survey of the recent dynamic corporate finance literature.

economy reaches its new steady state. This effect is in sharp contrast to a reform that only eliminates enforcement frictions. In this case, aggregate output immediately rises by about 1.5 percent after the reform and decreases by about 0.5 percent in the subsequent 30 years. Third, although eliminating private information frictions yields significant welfare gains, eliminating limited enforcement frictions in an economy with private information does not. Fourth, there are significant complementarities between different types of financial reforms. The effect of a reform that reduces both the enforcement and private information frictions yields aggregate welfare gains that are, on average, between 1.7 and 2.2 percentage points higher than the combined aggregate welfare gains associated with the implementation of individual reforms of the same magnitude. Fifth, a substantial fraction of these gains stems from changes in the structure of the financial contract rather than cost savings on monitoring and enforcement.

One of the main mechanisms driving these aggregate results is that, in the presence of private information frictions, limited enforcement frictions only affects the smallest and most inefficient firms in the economy. Consequently, a reform that improves contract enforcement relaxes the financial constraints on these small firms and only has a limited effect on aggregate resource allocation and welfare. When private information frictions are reduced, the economy grows, as a large mass of small incumbents slowly converges to the new stationary distribution in which firms are, on average, larger, demand more labor, and pay a higher wage. In addition, complementarities between the two types of reforms arise because young firms are on average larger when enforcement capacity is high. Because young entrepreneurs can build a greater stake in their firm more rapidly with high enforcement, the moral hazard problem becomes less significant.

Taken together, these results provide new answers to a number of questions regarding the aggregate effects of financial frictions. Regardless of the form of financial frictions, financial development leads to firms that, on average, are larger and grow slower. Thus, although the cross-country heterogeneity in firm growth noted by [Arelano et al. \[2012\]](#) could be driven by cross-country differences in contract enforcement, the analysis suggests that it could also be driven by cross-country differences in the efficacy of the monitoring technology as emphasized in [Greenwood, Sanchez, and Wang](#)

[2010] and Greenwood et al. [2013]. Moreover, the quantitative results are consistent with Midrigan and Xu [2014] insofar as the misallocation associated with limited enforcement conditional on the effect of private information is small, as firms can rapidly grow out of the borrowing constraints implied by limited enforcement. However, the effect of private formation on misallocation is much greater than that of limited enforcement, as the incentive compatibility constraint continues to bind for larger firms. Lastly, the analysis contributes a nuanced answer to the seminal question posed by Goldsmith [1969] as to whether financial development causes economic growth. In the model, financial development causes significant economic growth if it is associated with a reduction of private information frictions. However, the effects of financial development on economic growth are more modest if a reform that improves contract enforcement fails to address private information frictions.

The rest of the paper proceeds as follows. Sections 2 and 3 present the environment and the benchmark optimal financial contract, respectively. Section 4 defines the recursive stationary general equilibrium. Section 5 qualitatively analyzes the effect of financial reforms on the optimal contract. Section 6 discusses the parameterization of the benchmark model economy and the main results from the quantitative analysis. Section 7 concludes. Proofs of propositions that are essential to the analysis are collected in the Appendix. Additional details related to proofs and numerical strategies are relegated to the online appendix.

## 2 Model

Time is discrete and infinite, and each period is indexed by  $t$ . The economy is populated by continuums of entrepreneurs and workers. Agents' career path is fixed. Workers have an infinite lifetime, while entrepreneurs have an uncertain lifetime. Long-lived firms managed by entrepreneurs use labor supplied by workers and capital to produce a numéraire good used for consumption and investment.

## 2.1 Workers

There is a mass  $\lambda$  of workers endowed with one unit of time each period. Workers allocate their time between work and leisure, and they discount the future at rate  $(1+r)^{-1}$ , where  $r$  is the real interest rate.<sup>6</sup> Workers choose consumption  $c_t$  and labor  $l_t \in [0, 1]$  to maximize the value of their lifetime expected utility<sup>7</sup>

$$E_0 \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t [c_t - \varphi(l_t)] , \quad (1)$$

subject to the period budget constraint

$$c_t + d_{t+1} \leq (1+r)d_t + w_t l_t , \quad (2)$$

and  $d_{t+1} \geq -\underline{d}$ , where  $w_t$  is the wage rate,  $d_t$  is the asset position at time  $t$ , and  $\underline{d}$  is a natural borrowing limit. The function  $\varphi(\cdot)$  is strictly convex and satisfies  $\varphi(0) = 0$ .

## 2.2 Entrepreneurs

There is a unit mass of entrepreneurs born without wealth that survive into the next period with fixed probability  $(1-\gamma)$  and are replaced by new ones upon death. Entrepreneurs are risk-neutral and discount the future at rate  $(1-\gamma)/(1+r)$ . Entrepreneurs choose their consumption to maximize the value of their lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \left( \frac{1-\gamma}{1+r} \right)^t c_t . \quad (3)$$

Entrepreneurs do not participate in the labor market and may start a long-lived firm to generate consumption.

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<sup>6</sup>I am anticipating that the real interest rate  $r$  is constant in equilibrium.

<sup>7</sup>The assumptions on the workers are borrowed from [Cooley et al. \[2004\]](#).

## 2.3 Technology and firms

Long-lived firms can be built according to a blueprint technology and operated by entrepreneurs. There is one type of blueprint in the economy, and every entrepreneur not already operating a firm has access to one. The blueprint is a plan to set up a firm with decreasing returns to scale technology  $\tilde{f}(k, n)$ , where  $k$  and  $n$  are capital and labor inputs, respectively. The production function is strictly increasing in labor and capital, strictly concave, and satisfies  $\tilde{f}(k, 0) = \tilde{f}(0, n) = 0$ . Starting a firm requires paying an initial fixed cost  $I_0$ , which is partially sunk as firms can always be liquidated with scrap value  $S$  such that  $0 < S < I_0$ .

Once started, a firm requires working capital  $R$  each period that is used to rent capital  $k$  and to hire labor  $n$  at wage  $w$  before production can take place. The capital used in production depreciates at rate  $\delta \in (0, 1)$ . Firm cash flow is subject to a sequence of independent and identically distributed idiosyncratic shocks  $(\nu_t)_{t \geq 0}$ , where  $P(\nu = 1) = 1 - P(\nu = 0) = p$ . It follows that the maximum expected cash flow generated with working capital  $R$  and the undepreciated capital  $(1 - \delta)k(R)$  left after the previous period's production is given by

$$pf(R) + (1 - \delta)k(R) := \max_{k, n} p\tilde{f}(k, n) + (1 - \delta)k \quad (4)$$

s.t.  $k + wn \leq R$ .

The assumptions about the firm technology imply there exists a unique level of working capital  $\tilde{R}$  that maximizes expected profit, such that

$$\tilde{R} := \operatorname{argmax}_R pf(R) + (1 - \delta)k(R) - (1 + r)R \quad (5)$$

for a given interest rate  $r$  and wage rate  $w$ . The assumption of diminishing returns to scale with fixed start-up costs implies that it can be optimal to organize production around large firms. A firm is terminated upon the death of its entrepreneur, which occurs with probability  $\gamma$  and is analogous to receiving a permanent zero-productivity shock.<sup>8</sup> It is profitable to start a firm only if the expected discounted lifetime profit of

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<sup>8</sup> This assumption captures other sources of firm exit that are not modeled explicitly and allows

a firm operating at full scale is greater than the cost of starting the firm—that is,

$$\frac{\Pi(\tilde{R})}{1 - \beta} > (1 + r)I_0, \quad (6)$$

where  $\beta = (1 - \gamma)/(1 + r)$  and  $\Pi(R) = pf(R) + (1 - \delta)k(R) - (1 + r)R$  is defined as the period expected profit achievable with resources  $R$ . Lastly, entrepreneurs can only manage one firm, and starting a new firm requires an entrepreneur to abandon her current firm.

## 2.4 Financial intermediation and financial frictions

Financial intermediaries are institutions that arise to provide workers and entrepreneurs a technology to save and borrow. Financial intermediaries offer long-term contracts to entrepreneurs to finance their firms in exchange for payments. Financial intermediaries can fully commit to these contracts, but entrepreneurs operating their firms under limited liability may repudiate their debt. In case of repudiation, an entrepreneur absconds with the period cash flow, loses the firm, and may enter into a new contract with another intermediary to start a new firm. The diverted cash flow cannot be invested in a new project.

The value of repudiating a contract is the sum of the period random cash flow  $\nu \cdot f(R)$ , which is non-decreasing in working capital  $R$ , and the external value of setting up a new project net of any repudiation costs denoted by  $\mathcal{O}$ , which is determined in general equilibrium. In addition to limited contract enforcement, entrepreneurs have private information about  $\nu$  and may misreport the period cash flow to increase their consumption. The combination of limited enforcement and private information under limited liability leads to financial frictions that affect contracting with entrepreneurs.

Payments from the entrepreneurs to the financial intermediary are used to repay the outstanding debt and to accumulate deposits. Financial intermediaries use deposits from workers and entrepreneurs to finance their portfolio of firms. One unit of workers'

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me to pin down the steady-state distribution of firms without keeping track of individual firms—see, for instance, Cooley and Quadrini [2001], Cooley et al. [2004], and Smith and Wang [2006].

deposits at the beginning of a period pays  $1 + r$  units at the end of the period. Because the deposits accumulated by deceased entrepreneurs are available to finance firms in an intermediary's portfolio, the rate of return on entrepreneurs' deposits is  $(1 + r)/(1 - \gamma)$ . Perfect competition and free entry in the financial intermediation industry imply that any coalition of entrepreneurs can establish a financial intermediary. In equilibrium, who owns a financial intermediary is irrelevant because of zero profit. Consequently, I focus on a single representative financial intermediary in the remainder of the paper.

### 3 Optimal financial contracting in the benchmark model economy

In the benchmark economy, I assume that a financial intermediary cannot monitor firm cash flows and cannot invest in enforcement capacity to decrease  $\mathcal{O}$ . An interpretation of this benchmark is an economy in which the costs of investing in firm monitoring and enforcement capacity are so high that they are never incurred by the intermediary. These high costs could reflect, for example, the lack of institutional and legal infrastructures necessary to support consistent corporate financial reporting and recourse lending and to exclude defaulting entrepreneurs from capital markets. This benchmark contract can be interpreted as the union of the long-term contract optimal under private information studied in [Quadrini \[2004\]](#) and [Clementi and Hopenhayn \[2006\]](#) and the long-term contract optimal under limited enforcement studied in [Albuquerque and Hopenhayn \[2004\]](#).

The financial intermediary has the same discount rate as entrepreneurs and offers them a state-contingent financial contract that induces truthful reporting and debt repayment each period. Denote the reporting strategy of an entrepreneur by the sequence of reports  $\hat{\nu} = (\hat{\nu}_t(\nu^t))_{t \geq 0}$ , where  $\nu^t = (\nu_1, \dots, \nu_t)$  is the true history of the cash flow shocks experienced by the entrepreneur up to period  $t$ . The history of reports up to period  $t$  is denoted by  $h^t = (\hat{\nu}_1, \dots, \hat{\nu}_t)$ . A financial contract  $\sigma = \{\ell_t(h^{t-1}), Q_t(h^{t-1}), R_t(h^{t-1}), \tau(h^{t-1}, \hat{\nu}_t)\}$  specifies a liquidation rule  $\ell_t(h^{t-1}) \in \{0, 1\}$ , transfer  $Q_t(h^{t-1}) \in \mathbb{R}_+$  from the intermediary to the entrepreneur in the event of liq-

liquidation, period working capital  $R_t(h^{t-1}) \in \mathbb{R}_+$ , and transfers  $\tau(h^{t-1}, \widehat{\nu}_t) \in \mathbb{R}$  between the entrepreneur and the financial intermediary conditional on the ex-post cash flow report  $\widehat{\nu}_t$ .

The timing within a period is as follows. At the start of the period, the intermediary decides whether the firm should be liquidated. If the firm is liquidated, the intermediary transfers  $Q_t(h^{t-1})$  to the entrepreneur and recoups  $S - Q_t(h^{t-1})$ . If the firm is not liquidated, working capital  $R_t(h^{t-1})$  is loaned to the entrepreneur to hire labor and rent capital before production takes place. After the period cash flow is realized, the entrepreneur decides whether to repudiate the contract. If she does not repudiate the contract, the entrepreneur makes a transfer  $\tau(h^{t-1}, \widehat{\nu}_t)$  to the intermediary based on her cash flow report  $\widehat{\nu}_t$ . At the end of the period, the firm either permanently exits with probability  $\gamma$  or remains in operation. Figure 1 summarizes the sequence of events within one period.

[FIGURE 1 ABOUT HERE]

### 3.1 Recursive formulation

After history  $h^{t-1}$ , the pair of contract and report strategy  $(\sigma, \widehat{\nu})$  implies an expected discounted cash flow  $V_t(\sigma, \widehat{\nu}, h^{t-1})$  and  $B_t(\sigma, \widehat{\nu}, h^{t-1})$  for the entrepreneur and the financial intermediary, respectively. Following Green [1988], Spear and Srivastava [1987], and others, the contract can be solved recursively using  $V_t$  and  $V_{t+1}$  as the continuation values awarded to the entrepreneur contingent on her reports. Following Quadrini [2004], Albuquerque and Hopenhayn [2004], and Clementi and Hopenhayn [2006], a feasible and incentive-compatible contract is optimal if it maximizes  $B_t(V_t)$  for every feasible  $V_t$ . This maximization problem is analogous to maximizing the value of the joint surplus  $W_t(V_t) = B_t(V_t) + V_t$ , which can be interpreted as the value of the firm by interpreting  $V_t$  and  $B_t(V_t)$  as equity and debt, respectively.

Let  $V^L$  and  $V^H$  denote the promised continuation values awarded to an entrepreneur reporting low and high cash flow, respectively. Because the optimal contract induces truth-telling, I do not distinguish between the cash flow report  $\widehat{\nu}(\nu)$  and the actual cash

flow realization  $\nu$  unless otherwise necessary. The optimal contract must satisfy five constraints. First, feasibility requires that the transfers from the entrepreneur to the intermediary should not exceed the period cash flow, as entrepreneurs do not have access to an alternative saving technology. Referring to  $\tau$  as the transfer to the intermediary after a high report and assuming that the firm efficiently allocates  $R$ , the feasibility constraint simplifies to

$$\tau \leq f(R) , \quad (7)$$

where  $f(R)$  is the cash flow generated by a firm with access to working capital  $R$  conditional on a high shock. Second, the value  $V_C$  promised to an entrepreneur if the firm is not liquidated should be equal to the expected period cash flow net of repayment plus the discounted expected promised continuation value:

$$V_C = p(f(R) - \tau) + \beta(pV^H + (1 - p)V^L) . \quad (8)$$

Third, incentive compatibility requires that following a high report, transfers from the entrepreneur plus the discounted high promised continuation value,  $V^H$ , be no less than the value of diverting and immediately consuming the period cash flow and receiving a low continuation value  $V^L$  next period:

$$f(R) - \tau + \beta V^H \geq f(R) + \beta V^L . \quad (9)$$

Fourth, debt repayment requires that the value derived from repaying the debt be greater than the value of diverting the period cash flow and pursuing another opportunity denoted by  $\mathcal{O}$ , which is determined in general equilibrium and will be defined in the next section. Given that the incentive compatibility constraint holds, debt repayment requires that

$$\beta V^L \geq \mathcal{O} . \quad (10)$$

Fifth, firms operate under limited liability that requires that

$$V^L \geq 0 \text{ and } V^H \geq 0 . \quad (11)$$

Conditional on the entrepreneur being alive, the intermediary can recover  $S$  by liquidating the firm at the start of a period. The highest joint surplus can be achieved by randomizing the liquidation decision such that

$$\begin{aligned} W(V) = \max_{\alpha \in [0,1], Q, V_C} & \quad \alpha S + (1 - \alpha) \widehat{W}(V_C) \\ \text{s.t.} & \quad \alpha Q + (1 - \alpha) V_C \geq V, \end{aligned} \tag{12}$$

where  $\alpha$  is the probability that the firm is liquidated. If the firm is not liquidated with probability  $1 - \alpha$ , the entrepreneur receives continuation value  $V_C$ , and the value of the joint surplus conditional on the firm avoiding liquidation is given by

$$\begin{aligned} \widehat{W}(V) = \max_{\tau, R, V^L, V^H} & \quad \Pi(R) + \beta(pW(V^H) + (1 - p)W(V^L)) \\ \text{s.t.} & \quad (7), (8), (9), (10), \text{ and } (11), \end{aligned} \tag{13}$$

where  $\Pi(R) = pf(R) + (1 - \delta)k(R) - (1 + r)R$  is the maximum period expected profit achievable with resources  $R$ .

### 3.2 Properties of the benchmark contract

Assume for the moment that the prices and the outside option value  $\mathcal{O}$  are constant. After the contract is signed, the intermediary finances the set-up cost  $I_0$  and the initial working capital  $R_0$ . Because entrepreneurs with zero net worth have an incentive to misreport the cash flow and repudiate their debt, the initial loan  $R_0$  is less than the working capital needed by the firm to operate at its efficient scale,  $\widetilde{R}$ . The evolution of working capital  $R(V)$  depends on the evolution of the entrepreneur stake in the project  $V$ , which is determined by the optimal contract.

Consider first the optimal contract with  $\mathcal{O} = 0$ . Under this assumption, an entrepreneur who repudiates her debt diverts and consumes the cash flow  $f(R)$  and loses access to the capital market in the subsequent periods. This case is similar to the contract studied by [Clementi and Hopenhayn \[2006\]](#).<sup>9</sup> Under risk neutrality, it is optimal to

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<sup>9</sup>To be sure, [Clementi and Hopenhayn \[2006\]](#) do not consider limited enforcement in their model

set repayment such that  $\tau = f(R(V))$  whenever  $V^H(V) \leq \tilde{V}$ , as it allows for the fastest accumulation of equity toward the unconstrained level.<sup>10</sup> The incentive compatibility constraint simplifies to

$$\beta V^H \geq f(R) + \beta V^L . \quad (14)$$

Combining the incentive compatibility and promise-keeping constraint implies that the promised continuation values evolve according to  $\beta V^L(V) = V - pf(R(V))$  when  $V \leq \tilde{V}$  and  $\beta V^H(V) = V + (1 - p)f(R(V))$  when  $V^H(V) \leq \tilde{V}$ —that is, the intermediary optimally decreases an entrepreneur’s value after low cash flow and increases her value otherwise, and  $V^L < V < V^H$  for all  $V^H(V) \leq \tilde{V}$ . Conditional on not exiting exogenously, the firm grows on average when  $V^H(V) \leq \tilde{V}$ , as  $E(V'|V) = pV^H + (1 - p)V^L = V/\beta > V$  for all  $V^H(V) \leq \tilde{V}$ . Feasibility and incentive compatibility imply that the period working capital  $R(V)$  is determined by  $f(R(V)) \leq \beta(V^H(V) - V^L(V))$ . [Clementi and Hopenhayn \[2006\]](#) show that  $R(V)$  is increasing in  $V$  for  $V < \tilde{V}$  except in the neighborhood of the liquidation region, and that a constrained entrepreneur always receives less than the efficient level of working capital,  $R(V) < \tilde{R}$  for all  $V < \tilde{V}$ . The firm either reaches the unconstrained level  $\tilde{V}$  after experiencing a sufficiently long but finite sequence of high-cash flow shocks or faces liquidation with a positive probability whenever  $V$  falls below a threshold  $V_R$ . Whenever  $V$  falls below  $V_R$ , the firm is liquidated with probability  $\alpha(V) = (V_R - V)/V_R$ , in which case it is optimal for the intermediary to set  $Q = 0$ . If the firm survives the liquidation lottery with probability  $1 - \alpha(V)$ , its entrepreneur is awarded the continuation value  $V_C = V_R$ . If an entrepreneur’s value  $V$  reaches the value  $\tilde{V} = pf(\tilde{R})/(1 - \beta)$ , any agency or enforcement problem becomes irrelevant, as the entrepreneur has no debt and has accumulated enough deposit to self-finance her firm at the efficient level  $\tilde{R}$  in all subsequent periods.

When  $\mathcal{O} > 0$ , the enforcement constraint imposes a lower bound on the expected continuation values. In this case, the enforcement constraint restricts the range of values

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with private information. However, an enforcement constraint is implicitly satisfied under limited liability when  $\mathcal{O} = 0$ .

<sup>10</sup>To see this, note that setting  $\tau = f(R)$ , the participation constraint implies that  $V/\beta = pV^H + (1 - p)V^L$ , so that the entrepreneur’s value grows at the maximum feasible rate.

$V^H(V)$  and  $V^L(V)$  the intermediary can use to discipline the moral hazard.<sup>11</sup> To see this effect, note that preventing default requires that  $pV^H(V) + (1-p)V^L(V) \geq pf(R) + \mathcal{O}$ . A higher  $\mathcal{O}$  implies a lower  $R(V)$  for a given  $V < \tilde{V}$ , as  $f(R(V)) \leq \beta(V^H(V) - V^L(V))$  must hold to maintain incentive compatibility. This implies the joint surplus  $\widehat{W}(V)$  is also lower for all  $V < \tilde{V}$ . In other words, a higher outside value  $\mathcal{O}$  requires the entrepreneur to build a greater stake in her firm to obtain the same level of financing until she becomes unconstrained. A key difference relative to the contract with  $\mathcal{O} = 0$  is that there will be no liquidation in equilibrium if the liquidation threshold is such that  $V_R < \mathcal{O}$ , as it is not feasible to promise continuation values that are less than  $\mathcal{O}$ .<sup>12</sup>

## 4 Recursive stationary general equilibrium

There are three endogenous prices in the model economy: the entrepreneur's outside option  $\mathcal{O}$ , the interest rate  $r$ , and the wage rate  $w$ . Under the assumptions that workers' instantaneous utility is linear in consumption, the capital market trivially clears given a pair of values for  $\mathcal{O}$  and  $w$ . The remainder of this section explains how  $\mathcal{O}$  and  $w$  are determined in general equilibrium.

The entrepreneur's outside option  $\mathcal{O}$  is defined as

$$\mathcal{O} = (1 - \kappa)\beta V_0, \quad (15)$$

where  $\beta V_0$  is the value an entrepreneur can derive by starting a new firm in the next period,  $\kappa\beta V_0$  is the cost of repudiating the contract, and  $\kappa \in (0, 1)$  is a fixed parameter whose value determines the feasibility of the contract. It is clear that the initial entrepreneur's value  $V_0$  enters the dynamic contracting problem (12) and (13) via the

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<sup>11</sup>Imposing a lower bound on the continuation value in this problem has been studied by [Quadrini \[2004\]](#) in the context of renegotiation-proof contracts. Although [Quadrini \[2004\]](#) does not consider limited enforcement, imposing a lower bound on expected continuation values is analogous to imposing an enforcement constraint if the lower bound is interpreted as the entrepreneur outside value. Consequently, proofs of the results established by [Quadrini \[2004\]](#) are not repeated.

<sup>12</sup>Because the liquidation cutoff  $V_R$  satisfies  $\widehat{W}'(V_R) = (\widehat{W}(V_R) - S)/V_R$ , it follows that a lower  $\widehat{W}'(V)$  for all  $V < \tilde{V}$  implies that  $V_R$  is larger. However, whether the increase in the liquidation threshold leads to an increase in the liquidation rate depends on the size of  $\mathcal{O}$ .

enforcement constraints. Because there is perfect competition in the financial intermediation sector,  $V_0$  must be the greatest feasible value such that the representative financial intermediary makes zero profit on a new contract. For given prices  $r$  and  $w$  and given the dynamic contracting problems (12) and (13), define the mapping  $\mathcal{T} : [0, \tilde{V}] \rightarrow [0, \tilde{V}]$  such that:

$$\mathcal{T}(V_0) = \sup_V \{V : W(V; V_0) - V = (1 + r)I_0\} , \quad (16)$$

where  $W(V; V_0)$  is the optimal value function corresponding to (12) and (13) given an initial firm value  $V_0$ . Lemma 1 below shows that this mapping admits a unique fixed point. In what follows,  $W(V)$  always refers to the optimal value function given  $V_0$  is the fixed point of the mapping  $\mathcal{T}$ .

**Lemma 1.** *When  $\kappa$  is not too low, the mapping  $\mathcal{T}$  admits a unique fixed point.*

The initial value  $V_0$  determines a new firm's working capital  $R(V_0)$  and total debt  $R(V_0) + I_0$ . The evolution of  $V$  depends on the properties of the optimal contracts and the sequence of independent random shocks for liquidation, cash flow, and exit. Consider the stochastic process  $(V_t)_{t \geq 0}$  corresponding to a firm that is indefinitely replaced by a new one upon exit. Because there is a unit mass of entrepreneurs in the economy, the distribution implied by  $(V_t)_{t \geq 0}$  is the distribution of firms. Denote by  $\mathcal{B}(\mathbf{V})$  the Borel  $\sigma$ -algebra on the state space of entrepreneur values  $\mathbf{V}$  and  $(\mathbf{Z}, \mathcal{Z})$  the probability measure space for the sequence of independent random shocks  $\epsilon = (\epsilon_t)_{t \geq 0}$  from shock space  $\mathbf{Z}$ . Appendix A.1 shows that  $(V_t)_{t \geq 0}$  is a time-homogeneous Markov chain on a general state space and its law of motion can be described by a measurable function  $F : \mathbf{V} \times \mathbf{Z} \rightarrow \mathbf{V}$ , a joint distribution  $\chi \in \mathcal{P}(\mathbf{Z})$ , where  $\mathcal{P}(\mathbf{Z})$  denotes the distributions on  $\mathbf{Z}$ , and an initial state  $V_0 \in \mathbf{V}$ , such that

$$V_{t+1} = F(V_t, \epsilon_t), \text{ and } \epsilon \sim \chi , \quad (17)$$

with its associated stochastic kernel given by

$$P(x, A) = \int \mathbf{1}_A[F(x, \epsilon)]\chi(d\epsilon) \quad (x \in \mathbf{V}, A \in \mathcal{B}(\mathbf{V})) . \quad (18)$$

Proposition 1 shows that, for given prices  $r$  and  $w$ ,  $(V_t)_{t \geq 0}$  has a stationary distribution  $\psi \in \mathcal{P}(\mathbf{V})$  that can be reached from any initial distributions in  $\mathcal{P}(\mathbf{V})$  as the number of periods tends to infinity, where  $\mathcal{P}(\mathbf{V})$  denotes the distributions on  $\mathbf{V}$ . Proposition 1 also establishes the parametric continuity of  $\psi$  using the results of [LeVan and Stachurski \[2007\]](#).

**Proposition 1.** *For given prices  $r$  and  $w$ , there exists a stationary distribution of firms  $\psi$  that is ergodic. Moreover,  $\psi$  is continuous in  $r$  and  $w$ .*

At any point in time, the representative financial intermediary holds a portfolio of contracts indexed by  $V$  and aggregate entrepreneur net-deposit is

$$D_e = - \int B(V) d\psi , \quad (19)$$

where  $B(V) = W(V) - V$ , which could be positive or negative depending on the shape of the distribution of firms  $\psi$ . Because entrepreneurs rent capital from the intermediary, the value of an unconstrained contract is  $B(\tilde{V}) = -[(r+\delta) \cdot k(\tilde{R}) + (1+r) \cdot w \cdot n(\tilde{R})]/(1-\beta)$ , which is negative. Therefore, a positive  $D_e$  implies that the representative intermediary uses deposits from entrepreneurs managing larger firms to finance smaller firms. A negative  $D_e$  implies that the representative intermediary must raise additional deposits  $D_w$  from workers to finance the portfolio of firms. It follows that the capital market clears when the net deposits from workers  $D_w$  and entrepreneurs  $D_e$  are just enough to finance the aggregate working capital and the set-up cost of new firms. That is,

$$D_e + D_w = \int R(V) d\psi + \hat{\gamma}(I_0 - S) + \gamma I_0 , \quad (20)$$

where  $\hat{\gamma}$  is the fraction of firms that are liquidated each period.

The labor market clears when labor demand from firms is equal to labor supplied by the workers, such that

$$L = \int n(V) d\psi , \quad (21)$$

where the wage rate is determined competitively by  $\varphi'(L) = w$ . Given that the labor and the capital market clear, the goods market also clears and aggregate output is divided

between worker and entrepreneur consumption and aggregate capital investment.<sup>13</sup> The definition of a recursive stationary competitive equilibrium for the economy is given below.

**Definition 1.** *A recursive stationary competitive equilibrium consists of prices  $w$  and  $r$ , an initial contract value  $V_0$ , worker labor supply and consumption function  $l(d)$  and  $c(d)$ , a contract  $\{R(V), \tau(V), V^L(V), V^H(V), \alpha(V), Q, V_C\}$ , and a stationary distribution  $\psi$  of entrepreneur values  $V$ , such that*

1. *the labor and consumption functions maximize workers' lifetime utility given in (1);*
2. *the financial contract maximizes the value of the firm according to (12) and (13);*
3.  *$V_0$  is such that the intermediary breaks even on new contracts and it is the fixed point of  $\mathcal{T}$ ; and*
4. *the labor and capital markets clear according to (21) and (20).*

Under the assumption of the worker's preferences, proposition 2 establishes the existence and uniqueness of the recursive stationary equilibrium.

**Proposition 2.** *There exists a unique recursive stationary competitive equilibrium.*

## 5 The effects of financial reforms on optimal financial contracting

An interpretation of the benchmark is a financial contract in which the costs of investing in enforcement capacity and monitoring firms are so high they are never incurred in equilibrium by the intermediary. This section considers the effects of reforms designed to improve contract enforcement and reduce private information by reducing the cost of investing in enforcement capacity and the cost of monitoring firms, respectively. To proceed, I specialize the financial contract by making two additional assumptions.

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<sup>13</sup>See the Appendix for more details.

First, I assume that the intermediary can monitor firm cash flow by incurring a fixed cost  $m$  at the beginning of each period. Paying  $m$  lets the intermediary verify the authenticity of a cash flow report but does not let him recover any diverted cash flow in the event that the entrepreneur misreports. Second, following Popov [2014], I assume that the intermediary can invest  $\phi(e)$  at the beginning of a period to decrease the entrepreneur's outside value to  $\mathcal{O} - e$ , where  $e$  denotes the level of enforcement.<sup>14</sup> That is, although the intermediary cannot recover the diverted cash flow, he can make it more difficult for an entrepreneur to finance a new firm after repudiating the contract. The enforcement cost function  $\phi(\cdot)$  is strictly increasing, convex, and satisfies  $\phi(0) = 0$ . Financial reforms seek to decrease the enforcement cost, the monitoring cost, or both.

Assuming once more that repayments by constrained entrepreneurs are set optimally such that  $\tau = f(R)$ , the value of the joint surplus conditional on no liquidation and before deciding on monitoring is given by

$$\widehat{W}_i(V) = \max_{e, R, V^L, V^H} \Pi(R) - (1+r)(\phi(e) + \mathbf{1}_{\{i=M\}}m) + \beta(pW(V^H) + (1-p)W(V^L)) \quad (22)$$

$$\text{s.t. } V \geq \beta(pV^H + (1-p)V^L)$$

$$\beta V^H \geq f(R) + \beta V^L \text{ if } i = NM \quad (23)$$

$$\beta V^H \geq f(R) + \mathcal{O} - e \quad (24)$$

$$\beta V^L \geq \mathcal{O} - e, \quad (25)$$

where  $i \in \{M, NM\}$  and  $M$  and  $NM$  indicate *Monitoring* and *No Monitoring*, respectively. Regardless of monitoring, the enforcement constraints (24) and (25) must be satisfied. Therefore, the key difference between  $\widehat{W}_M(V)$  and  $\widehat{W}_{NM}(V)$  is the incentive compatibility (23) that must be satisfied when the intermediary does not pay  $m$  to monitor the firm. Under some conditions, and because of the concavity of the value function, it is optimal for the intermediary to randomize over the monitoring decision

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<sup>14</sup>Popov [2014] studies optimal enforcement decision in a dynamic lending contract with limited enforcement and full information.

such that

$$\widehat{W}(V) = \max_{\varsigma \in \{0,1\}, V_M, V_{NM}} \varsigma \widehat{W}_M(V) + (1 - \varsigma) \widehat{W}_{NM}(V) \quad (26)$$

s.t.  $\varsigma V_M + (1 - \varsigma) V_{NM} \geq V$ ,

where  $\varsigma$  is the probability that the intermediary commits at the beginning of the period to monitor the firm cash flow, and  $V_M$  and  $V_{NM}$  are the promised continuation values awarded to the entrepreneur given that the intermediary commits to monitor or not, respectively. Note that although  $\varsigma$  does not depend on the ex-post cash flow realization during the period, it does depend on the financial position  $V$  of the firm at the start of the period. The liquidation decision is defined as before in (12).

## 5.1 Improving contract enforcement

A financial reform seeking to improve contract enforcement reduces  $\phi(e)$  for all  $e > 0$ . When  $m \rightarrow \infty$ , as in the benchmark contract, the intermediary never chooses to monitor the firm in equilibrium. That is,  $\varsigma(V) = 0$  for all  $V \in [V_R, \tilde{V}]$  and  $\widehat{W}(V) = \widehat{W}_{NM}(V)$ .

When the enforcement constraint binds and  $\beta V^L = \mathcal{O} - e$ , greater investment in enforcement capacity reduces small entrepreneurs' incentive to default, which has two distinct effects on lending. First, investing in  $e$  acts as insurance against default. Second, a higher  $e$  also increases the range of continuation values the intermediary can use to discipline the moral hazard. To see this, rearrange the first-order conditions and the envelope condition to obtain<sup>15</sup>

$$f'(R) [\phi'(e)(1 + r) + (1 - p) (W'(V^L) - W'(V))] = \Pi'(R) . \quad (27)$$

Equation (27) shows that optimal enforcement  $e(V)$  and the period working capital  $R(V)$  are set such that, conditional on  $V$ , the marginal value of maintaining the credit relationship—the right-hand side of equation (27)—is equal to the marginal benefit of insuring the period cash flow against default plus the marginal benefit of relaxing the constraint on the low promised continuation value—the left-hand side of equation (27).

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<sup>15</sup>The first order conditions for this problem are reported in Appendix A.1.

Rearranging further yields,

$$W'(V) = (1 + r)\phi'(e) + [(1 - p)W'(V^L) + pW'(V^H)] \quad , \quad (28)$$

showing that by investing in enforcement capacity, the intermediary is willing to accept lower debt repayment from small firms reporting low cash flow in the current period because it can promise them continuation values that maintain incentive compatibility and debt repayment in the next period. This new mechanism is at the heart of the complementarities between reforms investigated in section 6.

Proposition 3 below summarizes the main properties of the optimal enforcement capacity investment under private information.

**Proposition 3.** *When the monitoring cost is such that  $m \rightarrow \infty$ ,  $V$  is sufficiently low, and the enforcement constraint binds:*

1.  $e(V) > 0$ ;
2. increasing  $e(V)$  permits lending a higher  $R(V)$  for a given  $V$ ; and
3.  $e(V)$  is nonincreasing in  $V$ .

Proposition 3 shows that the intermediary invests more in enforcement capacity when the entrepreneur's equity is lower and her incentive to repudiate the contract is higher. When the entrepreneur's equity value becomes larger than the outside value option—i.e., when  $V > \mathcal{O}$ —the intermediary stops investing in enforcement capacity and only disciplines the moral hazard by promising high and low continuation values that are contingent on the cash flow report. Given that  $V$  grows on average, proposition 3 implies that investment in enforcement also decreases with firm age.

From equation (27), a lower enforcement cost  $\tilde{\phi}'(e)$  such that  $\phi'(e) > \tilde{\phi}'(e)$  for all  $e > 0$  increases the optimal  $e(V)$  and decreases  $V_L(V)$ , which relaxes the constraint on  $R(V)$ . In the limit in which investing in enforcement is costless and  $\phi(e) = 0$  for all  $e > 0$ , the contract reduces to the one studied by Clementi and Hopenhayn [2006], as  $e$  can be set to  $\mathcal{O}$  at no cost.

## 5.2 Reducing private information

A financial reform seeking to reduce private information frictions reduces the monitoring cost  $m$ . Consider first the limiting case with  $m = 0$ , in which limited enforcement is the only relevant friction, as the intermediary always chooses to monitor the firm—that is,  $\varsigma(V) = 1$  for all  $V \in [V_C, \tilde{V}]$ , where  $\tilde{V}$  is the unconstrained equity value and  $W_M(V) = W(V)$ . If  $m = 0$  and the cost of investing in enforcement capacity  $\phi(\cdot)$  is not too high, the contract corresponds to a simple version of the one studied by Popov [2014].<sup>16</sup> If  $m = 0$  and investing in enforcement is very costly—i.e.,  $\phi(e) \rightarrow \infty$  for all  $e > 0$ —the contract is similar to the one studied by Albuquerque and Hopenhayn [2004], as  $e(V) = 0$  for all  $V$ .<sup>17</sup>

The entrepreneur’s promised continuation values are determined as follows. When cash flow reports can be verified, the intermediary optimally sets a misreporting entrepreneur’s promised continuation value to zero and ends the contract, as no other continuation value can increase the joint surplus. In this case,  $V^L(V)$  refers to the promised continuation value awarded to an entrepreneur that truthfully reports low cash flow and it is well-known that it is optimal to maintain this entrepreneur at the same size such that  $V^L(V) = V$  (Albuquerque and Hopenhayn [2004]). This implies that the incentive compatibility constraint is satisfied as long as the enforcement constraint holds when the cash flow can be verified. Furthermore, the property that  $V^L(V) = V$  and the participation constraint implies that  $V^H(V) = \mathbf{c}V$  for  $V^H(V) < \tilde{V}$  where  $\mathbf{c} = \frac{1-\beta p}{\beta(1-p)} > 1$  when  $\beta < 1$  so that an entrepreneur’s value increases after reporting high cash flow. Consequently, a firm may never face liquidation if the entrepreneur’s initial stake is greater than the liquidation threshold, such that  $V_0 > V_R$ , where  $V_R$  is the previously defined liquidation threshold. As in the benchmark contract, firms grow on average, as  $\mathbb{E}(V'|V) = p\mathbf{c}V + (1-p)V > V$  for all  $V^H(V) \leq \tilde{V}$ .

<sup>16</sup>A key difference is that the entrepreneur in the model studied by Popov [2014] is risk averse.

<sup>17</sup>A key difference between this contract and the contract studied by Albuquerque and Hopenhayn [2004] is that the entrepreneur in this model must finance the period working resources before production can take place.

The optimal period loan  $R(V)$  is

$$R(V) = \begin{cases} f^{-1}(\mathbf{c}V - (\mathcal{O} - e(V))) & \text{if } V < V_u < \tilde{V} \\ \tilde{R} & \text{if } V_u \leq V \leq \tilde{V} \end{cases}, \quad (29)$$

where  $V_u$  is such that  $f^{-1}(\mathbf{c}V_u - (\mathcal{O} - e(V))) = \tilde{R}$ . The value  $V_u$  is the minimum equity at which an entrepreneur can obtain the unconstrained level of financing from the intermediary and is generally less than the unconstrained level  $\tilde{V}$ . Although the firm operates at its efficient scale when  $V \in [V_u, \tilde{V})$ , the entrepreneur still requires financing from the intermediary until  $V$  reaches  $\tilde{V}$ .

Optimal enforcement  $e(V)$  is determined by

$$f'(R)\phi'(e)(1+r) = \Pi'(R), \quad (30)$$

which is the analog of equation (27) and shows that investing in enforcement capacity only acts as insurance against default when cash flow reports can be verified. As before, the tradeoff between the cost of enforcement and debt repayment is given by equation (28). Proposition 4 shows that it is optimal for the intermediary to invest in enforcement capacity  $e(V)$  as long as the firm operates below its efficient level, and  $e(V)$  is strictly decreasing in  $V$ . Moreover, proposition 4 implies that  $e(V)$  decreases with firm age, as  $V$  grows on average.

**Proposition 4.** *When the monitoring cost is such that  $m = 0$  and for all  $V < V_u$ :*

1.  $e(V) > 0$ ;
2. *increasing  $e(V)$  permits lending a higher  $R(V)$  for a given  $V$ ; and*
3.  *$e(V)$  is strictly decreasing in  $V$ .*

I now turn to the monitoring decision. When  $m > 0$  is not too high, the intermediary chooses to monitor the firm before advancing  $R(V)$  by comparing the conditional values function  $\widehat{W}_M(V)$  and  $\widehat{W}_{NM}(V)$ . Proposition 5 below shows there is a range of values for the monitoring cost  $m$  for which these conditional value functions intersect at two

points. It follows that the highest value of the joint surplus  $W(V)$  can be achieved by randomizing the monitoring decision around  $V^{low}$  and  $V^{high}$  as defined in problem (26).

**Proposition 5.** *There is a finite range of positive values for the monitoring cost  $m$  such that  $\widehat{W}_M(V)$  and  $\widehat{W}_{NM}(V)$  intersect at two points  $V^{low}$  and  $V^{high}$ , where  $V^{low} < V^{high} < \widetilde{V}$ ,  $\widehat{W}_M(V) > \widehat{W}_{NM}(V)$  for all  $V \in (V^{low}, V^{high})$ , and  $\widehat{W}_M(V) \leq \widehat{W}_{NM}(V)$  for all  $V \notin (V^{low}, V^{high})$ .*

General properties of the equilibrium monitoring thresholds around  $V^{low}$  and  $V^{high}$  are difficult to derive, and an exhaustive analysis of the contract with endogenous monitoring and enforcement is outside the scope of this paper. To make some progress in characterizing the optimal monitoring decision, I rely on a numerical exercise using the same parameter values as in the quantitative exercise of section 6. Moreover, I assume that the initial value of the contract  $V_0$ , which is determined in general equilibrium, is such that  $V^{low} < V_0 < V^{high}$  and concentrate on the monitoring decision around  $V^{high}$ .<sup>18</sup> Under this assumption, the value of a new entrepreneur does not decrease following a truthful low-cash flow report because the intermediary commits to monitoring—i.e.,  $\widehat{W}_M(V_0) > \widehat{W}_{NM}(V_0)$ —and thus never reaches  $V^{low}$ .

The life-cycle of a firm when the monitoring and enforcement costs are not too high is summarized by Figures 2 and 3, which are obtained from the numerical exercise. In equilibrium, a firm starts at  $V_0 < V_M$  and is always monitored. If the cost of investing in enforcement capacity  $\phi(\cdot)$  is not too high, the intermediary also chooses how much to invest according to equation (30). Conditional on not exiting exogenously at the end of the period, the firm grows on average, and proposition 4 shows that the intermediary invests less in enforcement capacity over time. Given a long enough sequence of high cash flow, the firm passes the threshold  $V_M$  and enters the monitoring randomization region. With probability  $1 - \zeta(V)$ , the entrepreneur continues with promised value  $V_{NM}$  in the next period. From this point, the intermediary does not monitor the firm and disciplines the moral hazard by promising high and low continuation values as in the benchmark contract. The intermediary does not invest in enforcement capacity either

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<sup>18</sup>Although this assumption is not without loss of generality, it was found to hold in all numerical exercises.

past this point because only the incentive compatibility constraint continues to bind when  $V > V_{NM}$ . If the firm experiences a long enough sequence of low-cash flow shocks, it may re-enter the monitoring randomization region, but the value of the entrepreneur can never decrease below  $V_M$ . With a long enough sequence of high cash flow shocks, and conditional on not exogenously exiting, the firm reaches the unconstrained level.<sup>19</sup>

[FIGURE 2 ABOUT HERE]

[FIGURE 3 ABOUT HERE]

## 6 Aggregate consequences of financial reforms

In this section, I use a combination of calibration and estimation to parameterize the benchmark model economy using data from Colombia in the 1980s and early 1990s and analyze the effects of different types of counterfactual financial reforms. This choice of benchmark is motivated by World Bank research and data availability. The World Bank has identified a number of legal and institutional factors in Colombia in the 1980s and early 1990s that prevented efficient firm monitoring and financial contract enforcement. For example, infrastructures facilitating consistent and accurate corporate financial reporting were lacking at that time, which prevented efficient firm monitoring.<sup>20</sup> In addition, the heavy bias against secured creditors during insolvencies, the lack of effective collection mechanisms for creditors, and the protracted insolvency proceedings contributed to inefficient enforcement of contracts.<sup>21</sup> Against this institutional

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<sup>19</sup> This version of the contract with stochastic monitoring is related to [Monnet and Quintin \[2005\]](#), who consider the role of costly state verification and stochastic monitoring in a dynamic lending contract with fixed firm size.

<sup>20</sup>Part of the reason was the absence of prerequisite qualification for registration as a public accountant and the multiple and often contradictory source of accounting standards [[World Bank, 2003](#)]. In addition, although the majority of businesses were legally required to appoint a *revisor fiscal* for conducting annual audits, the *revisor fiscal* was also legally responsible for the quality of its client company's internal control, thereby precluding an actual external audit [[World Bank, 2003](#)].

<sup>21</sup>For example, estimating in advance the degree of coverage granted by the collateral was almost impossible in Colombia because mortgages and pledges could only be recovered after the debtor's labor and tax claims were fully paid with proceeds [[World Bank, 2006](#)]. In addition, majorities made up of unsecured creditors and shareholders of the insolvent firm could often alter their priorities and modify the conditions of their claims without the secured creditors' consent [[World Bank, 2006](#)]. Moreover, prior to reforms in 1995, mandatory liquidations were handled by civil courts lacking capacity, and

backdrop, the availability of detailed establishment-level data constructed by [Eslava, Haltiwanger, Kugler, and Kugler \[2004, 2013\]](#) makes Colombia in the 1980s and early 1990s an ideal laboratory to study the effects of different types of financial reforms. I then simulate the model to investigate the effects of a series of counterfactual reform experiments designed to reduce private information and limited enforcement frictions.

## 6.1 Parameterization of the benchmark model economy

The model is parameterized as follows. The worker disutility from labor is

$$\varphi(h) = A_c \cdot h^{\frac{1+\eta}{\eta}}, \quad (31)$$

where  $\eta$  is the elasticity of labor and  $A_c$  is a scaling parameter. The firm production function is

$$\tilde{f}(k, n) = z \cdot (k^\xi n^{1-\xi})^\theta, \quad (32)$$

where  $z$  is a scaling parameter.

The interest rate  $r$  is set to 0.12, which is roughly the average real lending rate during the 1980s and early 1990s documented by [Bond, Tybout, and Utar \[2015\]](#) and implies a discount rate for the workers around 0.89. The mass of workers  $\lambda$  is normalized to 1 and  $A_c$  is chosen so that hours worked are 0.36 to be consistent with the average for Colombia around that time. Following [Atkenson, Khan, and Ohanian \[1996\]](#) and [Cooley et al. \[2004\]](#), I set  $\theta = 0.85$ . This value implies  $\xi = 0.26$  for unconstrained firms given that the labor share in Colombia in the 1980s and early 1990s is about 0.63. The workers' elasticity of labor is set to 1 in the baseline parameterization, and I investigate the sensitivity of the results to other values. Lastly, the parameter  $\kappa$  is chosen to ensure that the contract is feasible, and  $z$  is set to normalize the unconstrained level of working capital  $R(\tilde{V})$  in the benchmark economy to 1.

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business knowledge could last many years [[Giné and Love, 2010](#)]. A contributor to the significant delays in claim collection was that debtors in Colombian executory proceedings were not limited to raising only specific defenses [[World Bank, 2006](#)]. Lastly, the legal framework and financial information infrastructure were not sufficiently developed to support efficient credit reporting and sharing of creditor information [[Miller and Gadamilas, 2006](#)].

The remaining parameters are the capital depreciation rate  $\delta$ , the parameter  $p$  that governs the cash flow shock process, the fixed initial investment  $I_0$ , the salvage value  $S$ , and the exogenous exit rate  $\gamma$ . I first estimate  $\delta$  and  $p$  by exploiting the cross-sectional and time-series variations in firm-level data on the Colombian manufacturing sector between 1982 and 1992.<sup>22</sup> The parameters  $I_0$  and  $S$  have an important effect on the contract and on firm dynamics but do not have a direct counterpart in the data. To make some progress, I follow the recent dynamic corporate finance literature—e.g., [Hennesy and Whited \[2005\]](#) and [DeAngelo, DeAngelo, and Whited \[2011\]](#)—and estimate structurally  $I_0$ ,  $S$ , and  $\gamma$  using an SMM procedure that chooses model parameter values that set moments of artificial data simulated from the model as close as possible to corresponding real data moments, conditional on the other calibrated and estimated parameters. Identification of the parameters by SMM comes from the contract properties with respect to these parameters rather than cross-sectional and time-series variation in the data.<sup>23</sup>

Panels A through C of table 1 summarize the parameterization of the benchmark economy. Most of the entries are self-explanatory, but a few require an explanation. The value of  $p$  is the average parameter value of a two-point approximation of the distribution of firm-level total factor productivity estimated by [Eslava et al. \[2004\]](#), where the average and standard deviation are taken across the three-digit manufacturing industries. As explained in the online appendix, a useful property of the optimal contract that facilitates the SMM estimation is that it becomes invariant to the wage rate  $w$  if  $z$  is set so that  $R(\tilde{V}) = 1$ ,  $I_0$  is expressed as a fraction of the maximum joint surplus  $W(\tilde{V})$ , and  $S$  is expressed as a fraction of  $I_0$ . Using this normalization, the parameter estimate of  $I_0$  implies that the fixed investment cost is 11.2 percent of the maximum joint surplus, which corresponds to 6.8 percent of the efficient level of resources  $\tilde{R}$  and 10.7 percent of the value of an unconstrained firm’s capital stock  $k(\tilde{R})$ .

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<sup>22</sup>The data used to estimate the contract parameters are a subset of the database constructed by [Eslava et al. \[2004\]](#) from the Colombian Annual Manufacturing Survey for 1982 to 1998. The data set used in this analysis contains information on the capital stock (buildings/structures and machinery/equipment) and total factor productivity of establishments in 29 three-digit manufacturing sectors that were operating between 1982 and 1992. See the Appendix for details.

<sup>23</sup>I explain the details of the SMM estimation procedure and the choice of moments in the Online Appendix.

Similarly, the value for  $S$  implies that 3.8 percent of the initial fixed investment  $I_0$  is sunk. The  $t$ -statistics in panel C suggest that the simulated moments match the actual moments well and that the model can also account reasonably well for the mean of firms' investment-to-capital ratio that was not targeted by the SMM procedure.

[TABLE 1 ABOUT HERE]

## 6.2 Steady-state allocations and firm dynamics

Table 2 summarizes the comparative statics results from two counterfactual reform experiments that completely eliminate one of the two sources of financial frictions in the benchmark economy. These exercises assume that the reformed economies reached their new steady state, and their transition dynamics are investigated in the next sub-section. The top half of table 2 reports values for aggregate output, aggregate consumption, labor hours, and the wage rate in the benchmark and the two reformed economies expressed as percentages of corresponding first-best values. The bottom half of table 2 reports the endogenous firm liquidation rate, the size of a new and average firm measured by its capital stock divided by the unconstrained level, and the fraction of firms operating at their efficient scale.<sup>24</sup>

[TABLE 2 ABOUT HERE]

In the baseline specification ( $\eta = 1$ ), table 2 shows that financial frictions in the benchmark economy lead to a 15.6 percent loss in aggregate output relative to the potential first-best level. Although limited enforcement by itself leads to a 7 percent loss in aggregate output, the relative effect of limited enforcement in the presence of private information frictions is small. Namely, private information by itself leads to

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<sup>24</sup>Note that the entry/exit rate, which is the sum of the liquidation rate and the exogenous exit rate  $\gamma$ , is slightly higher than the value reported in panel C of table 1. This difference arises because the exogenous exit rate  $\gamma$  is estimated rather than calibrated, and the estimated liquidation rate in the (finite sample) SMM procedure is different from the (asymptotic) solution of the model. For instance, the SMM procedure uses moments computed from a finite number of artificial data sets that each contain the lifecycles of the same number of firms as in the actual data set. In contrast, when solving for the general equilibrium, I estimate the ergodic distribution of promise utilities.

a 15 percent output loss, or nearly all of the aggregate output loss in the benchmark economy.

This result stems from the property that the enforcement constraint under private information only binds for the smallest firms in the economy while the incentive constraint binds for all firms requiring external financing. Consequently, private information frictions create a larger distortion in terms of aggregate output by creating a greater mass of inefficient firms. In addition, section 5 showed that eliminating enforcement frictions leads to two effects. First, firms can obtain a greater level of financing for the same level of equity. Second, the intermediary can maintain poor-performing firms at a smaller size while still maintaining incentive compatibility. In general equilibrium, a reform that eliminates enforcement frictions without reducing private information frictions leads to a wider distribution of firms and a higher liquidation rate, which, together with a higher wage rate, only leads to a modest increase in steady aggregate output. In contrast, a reform that eliminates private information frictions leads to a substantial increase in the fraction of firms that can operate at their efficient scale.

[FIGURE 4 ABOUT HERE]

Figures 4 and 5 illustrate this mechanism further by summarizing the effects of each reform on firm dynamics in the benchmark and post-reform steady states. Figure 4 plots the mean and standard deviation of firm growth rates conditional on firm age, and Figure 5 plots the average firm size conditional on firm age. When private information frictions are eliminated, young firms grow faster on average, their growth is less volatile, and virtually all 20-year-old firms operate at their efficient scale. In contrast, the average 20-year-old firm operates at about 80 percent of its efficient scale in the steady state when enforcement frictions are eliminated.

[FIGURE 5 ABOUT HERE]

Lastly, the misallocation of resources is larger when the workers' labor supply is more elastic, corresponding to a higher value of  $\eta$ . This is because financial constraints bind more with a more elastic labor supply, as a higher wage rate decreases the working

resources available to constrained firms. This effect is more pronounced in an economy with a relatively large fraction of smaller firms. For example, aggregate output in an economy with private information is almost 20 percent less than the first-best allocation when  $\eta = 2$ .

### 6.3 Transition dynamics of the reformed economies

In this sub-section, I analyze the transition dynamics of the benchmark economy in the years following a reform that either eliminates limited enforcement frictions or eliminates private information frictions. In these exercises, a reform is unexpected and permanent. The reform is implemented at the end period  $t_0$ , and after the promised continuation values with distribution  $\psi_{t_0}$  are awarded to entrepreneurs. The optimal contract includes the distribution of continuation values  $\psi_t$  as a state variable and solves

$$\begin{aligned} W(V, \psi_t) = \max_{\alpha \in [0,1], Q, V_C} \quad & \alpha S + (1 - \alpha) \widehat{W}(V_C, \psi_t) \\ \text{s.t.} \quad & \alpha Q + (1 - \alpha) V_C \geq V, \end{aligned} \tag{33}$$

where

$$\begin{aligned} \widehat{W}(V, \psi_t) &= \max_{R, V^H, V^L} \Pi(R, \psi_t) + \beta(pW(V^H, \psi_{t+1}) + (1 - p)W(V^L, \psi_{t+1})) \\ \text{s.t.} \quad & V \geq \beta(pV^H + (1 - p)V^L) \\ & \beta V^H \geq f(R, \psi_t) + \beta V^L \text{ and } V^L \geq 0 \text{ **after the enforcement reform, or}** \\ & \beta V^H \geq f(R, \psi_t) + \mathcal{O}(\psi_{t+1}) \text{ and } V^L \geq \mathcal{O}(\psi_{t+1}) \text{ **after the private information reform}** \\ & \psi_{t+1} = \mathcal{H}(\psi_t), \end{aligned}$$

where  $\mathcal{H}$  is the law of motion for the firm distribution, which is consistent with market clearing in each period  $t \geq t_0$  and is taken into account by workers when solving their consumption decision problem.<sup>25</sup>

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<sup>25</sup>Proposition 2 guarantees the existence and uniqueness of a post-reform recursive stationary competitive equilibrium. Proving the existence of a recursive competitive equilibrium in the transition between steady states is more difficult and would require a generalization of the proof given in appendix A.1 for the recursive stationary competitive equilibrium. Although I do not provide a formal proof in this paper, a possible argument would consist of showing that, given a sequence of distribution  $(\psi_t)_{t > t_0}$  and a law of motion  $\mathcal{H}$  for the distribution, there exists a sequence of prices consistent with

Figure 6 plots the transition of aggregate output, the wage rate, and the aggregate investment-to-capital ratio expressed in percentage deviation from the benchmark economy level in the 30 years following each reform. The solid line indicates the economy’s response for the baseline specification with  $\eta = 1$ . The economy’s response for higher and lower value of  $\eta$  is indicated by the dotted and dashed lines, respectively.<sup>26</sup>

[FIGURE 6 ABOUT HERE]

In the baseline specification ( $\eta = 1$ ), eliminating private information frictions leads to roughly 10 consecutive years of growth at an average growth rate of about 1 percent per year. This period of aggregate output growth is associated with a higher level of capital accumulation in the 10 years following the private information reform. Note that because there is no technological change in the economy—the blueprints are the same—this sustained period of growth is entirely due to improvement in contracts and resource reallocation.

Eliminating private information leads to two offsetting effects. On the one hand, holding prices fixed, eliminating private information implies that the contract only needs to prevent repudiation. As discussed above, the changes in firm dynamics implies that the firm distribution shifts to the right and the average firm is larger, which increases the aggregate joint surplus. On the other hand, when prices can adjust, the increase in average firm size increases the demand for labor, which increases the equilibrium wage rate. The higher wage rate, in turn, decreases the size of unconstrained firms, which partly offsets the first effect because unconstrained firms produce a large fraction

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market clearing and for which the optimal value function of the contract is well-defined. This new sequence of prices generates a new sequence of distribution  $(\psi_t)_{t>t_0}$  and a new law of motion  $\mathcal{H}'$ , which imply a new sequence of market clearing prices. Then, given a Cauchy sequence of prices from a set of Cauchy sequences and a law of motion for the firm distribution, one must show that this mapping based on market clearing returns a Cauchy sequence of prices from the same set, and that the new Cauchy sequence of prices returned by this mapping is closer at every point in the sequence to the previous Cauchy sequence of prices. A fixed point of this mapping, if it exists, is an equilibrium law of motion for the distribution of firms which determines the sequence of market clearing prices. The numerical algorithm discussed in the online appendix conjectures this argument is valid, and its iterative convergence starting from the pre-reform stationary distribution and prices is suggestive that such an equilibrium transition path may exist. The online appendix contains a definition of the equilibrium along the transition path.

<sup>26</sup>Recall that the real interest rate  $r$  is constant in the transition because workers’ utility is linear in consumption.

of aggregate output.<sup>27</sup> When the supply of labor is more rigid ( $\eta = 0.5$ ), the private information reform has a relatively smaller effect on aggregate output because the expansion of constrained firms following the reform is associated with a relatively larger increase in the aggregate labor demand and wages, which reinforces the second effect.

Eliminating enforcement frictions without addressing private information has widely different consequences for the economy. Following the enforcement reform, aggregate output immediately rises by 1.5 percent, but about one third of this initial growth is lost over the course of the next 30 years. This overshooting is due to a third effect. Although a lower outside option value of default immediately increases financing to a large fraction of small constrained firms, it also lets the financial intermediary induce truth-telling by maintaining young and poor performing firms at a lower size than before the enforcement reform. As a result, the distribution of firms becomes wider with a greater mass of small firms in the years following the enforcement reform, which partially offsets the initial gains from the enforcement reform.

## 6.4 Welfare gains

Table 3 reports the welfare gains associated with each reform by comparing steady states and by taking into account the effect of transitional dynamics. Recalling that the representative financial intermediary can be owned by any coalition of entrepreneurs, the steady state welfare is the sum of workers' wealth plus the aggregate joint surplus generated by the firms in the economy minus the aggregate sunk cost associated with setting up new firms. That is,

$$\text{Steady-state welfare} := \frac{wL(w, \psi) - \varphi(L(w, \psi))}{r} + \int W(V, \psi) d\psi - \frac{\hat{\gamma}(I_0 - S) + \gamma I_0}{r}, \quad (34)$$

where  $\hat{\gamma}$  denotes the fraction of firms that are liquidated in each period. Similarly, aggregate welfare, taking into account the transition dynamics of the economy, is defined

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<sup>27</sup>The online appendix investigates further the general equilibrium effects on reallocation by computing the transition of aggregate output holding the wage rate fixed at its pre-reform level. The online appendix also plots the transition of the equilibrium aggregate hours and capital stock.

as

$$\begin{aligned} \text{Dynamic welfare} := & \sum_{\iota=t_0+1}^{\infty} \left( \frac{1}{1+r} \right)^{\iota} [w_{\iota}L(w_{\iota}, \psi_{\iota}) - \varphi(L(w_{\iota}, \psi_{\iota}))] \\ & + \int W(V, \psi_{t_0+1})d\psi_{t_0} - \sum_{\iota=t_0+1}^{\infty} \left( \frac{1}{1+r} \right)^{\iota} (\hat{\gamma}_{\iota}(I_0 - S) + \gamma I_0), \end{aligned} \quad (35)$$

where  $\psi_{t_0}$  is the distribution of promised values at the end of period  $t_0$  when the reform is implemented. Computing welfare gains consists of subtracting the benchmark steady-state welfare from these quantities computed for the two different reforms.

[TABLE 3 ABOUT HERE]

Table 3 shows that, regardless of whether one accounts for transitory effects, eliminating private information frictions delivers aggregate welfare gains of at least 5 percent, while eliminating enforcement frictions delivers aggregate welfare gains around zero and negative in some cases. Welfare gains tend to be higher when the workers' labor supply is less rigid ( $\eta = 2$ ) because a higher wage rate raises workers' income and increases the intermediary's borrowing capacity.

Focusing on the different groups of agents in the economy, table 3 shows that entrepreneurs tend to lose from financial reforms when abstracting from transitional dynamics. This result is due to entrepreneurial rent. Constrained firms in the benchmark economy are very profitable because there is a fixed number of blueprints in the economy that can only be turned into firms by entrepreneurs. When financial frictions are reduced, firms in the new steady state are on average larger and entrepreneurial rent is lower because firms must pay higher wages to workers, resulting in welfare loss for entrepreneurs.

Entrepreneurs do not necessarily lose when taking into account transition dynamics. Because the transition of the economy to the new, post-private information reform steady state is relatively slow, entrepreneurs enjoy relatively high rent in the first 10 years or so following the reform. This effect is less pronounced when eliminating enforcement frictions, as the reform has a more modest and immediate impact on the economy.

## 6.5 Financial reform complementarities

The properties of the contract discussed in section 5.1 suggest there may be complementarities between financial reforms. By investing in enforcement capacity, the intermediary is willing to accept lower debt repayment from small firms in the current period while promising them continuation values that maintain incentive-compatibility and debt repayment in the next period. Because young firms can obtain greater working capital, on average, when enforcement capacity is high, young entrepreneurs build more rapidly a greater stake in their firm, and the moral hazard become less significant sooner. This sub-section investigates the significance of these micro effects for the aggregate.

Panels A and B of table 4 report the welfare and consumption gains, respectively, stemming from different types of reforms relative to the benchmark economy. I consider six different financial reforms indexed by a monitoring and enforcement cost, each set to  $\infty$ , an intermediate value, and zero. I measure reform complementarities as the difference between the gains following a reform that jointly lowers both the enforcement and monitoring costs, and the sum of the gains following reforms that individually lower only the enforcement or the monitoring cost.<sup>28</sup>

[TABLE 4 ABOUT HERE]

Focusing on the baseline specification ( $\eta = 1$ ), panel A of table 4 shows that the welfare gains are between 1.7 and 2.2 percentage points higher when a reform jointly addresses private information and enforcement frictions. For example, the welfare gains from a reform that jointly sets  $\phi(e) = 0.5e^2$  and  $m = 0.005$  is 5.44 percent with  $\eta = 1$ , while the sum of the gains from individual enforcement and private information reforms of the same magnitude is  $-1.84 + 5.08 = 3.24$  percent. The results are broadly similar

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<sup>28</sup>This exercise requires solving for the steady state of six different economies in addition to the benchmark economy. Note that there are still limited enforcement frictions when the cost of investing in enforcement capacity  $\phi(e) = 0$  for all  $e > 0$ . In this case, a repudiating entrepreneur abscond with the cash flow and is permanently excluded from the capital market—i.e.,  $\mathcal{O} = 0$ . Moreover, the enforcement constraint in this case never binds with  $m \rightarrow \infty$  and only binds for very small firms when  $m \in \{0, 0.005\}$ . As a result, the economy with zero monitoring and enforcement cost is very close to first-best.

when focusing on aggregate consumption (panel B), with gains between 1 and 2.8 percentage point higher, depending on the magnitude of the reforms.

These complementarities are also reflected in the aggregate expenditures on monitoring and enforcement. For example, the sum of expenditures on monitoring and enforcement after a reform that jointly sets  $\phi(e) = 0.5e^2$  and  $m = 0.005$  in the baseline economy ( $\eta = 1$ ) is about 22 percent lower than the sum of the expenditures on monitoring in an economy with  $\phi(e) \rightarrow \infty$  and  $m = 0.005$  and the expenditures on enforcement in an economy with  $\phi(e) = 0.5e^2$  and  $m \rightarrow \infty$ .

## 6.6 Sources of efficiency gains

I conclude the quantitative analysis by investigating the sources of efficiency gains associated with each type of financial reforms. Namely, I investigate whether the improvement in efficiency derives mainly from eliminating the deadweight losses associated with the removal of the monitoring and enforcement costs, or if it follows from salient changes in the structure of the financial contract.

Starting from the steady state of the economy with  $\{m = 0.05, \phi(e) = 0.5e^2\}$ , I compare the aggregate consumption gains that arise from implementing the same allocation without incurring the monitoring and enforcement costs to those gains that arise from reducing the monitoring and enforcement costs to zero. That is, in the former, aggregate consumption is higher because the expenditure on monitoring—i.e.,  $\int_{V \in [V_R, V_M]} m d\psi$ —and enforcement—i.e.,  $\int \phi(e(V)) d\psi$ —is not wasted, while in the latter, aggregate consumption gains stem from changes in the structure of the optimal contract. I then repeat this calculation starting from the steady state of the economy with  $\{m = 0.05, \phi(e) \rightarrow \infty \forall e > 0\}$  and  $\{m \rightarrow \infty, \phi(e) = 0.5e^2\}$ , respectively.

Panel C of table 4 summarizes the results and shows that a large fraction of the gains that arise from increasing enforcement or reducing private information stems from changes in the structure of the contracts. For example, in the baseline specification ( $\eta = 1$ ), aggregate consumption in an economy with no monitoring,  $m \rightarrow \infty$ , and some enforcement,  $\phi(e) = 0.5e^2$ , is 0.57 percent higher when the same allocation is attained without incurring the enforcement cost. In contrast, eliminating enforcement

frictions in this economy—i.e., setting  $\phi(e) = 0$ —raises aggregate consumption by about 1.47 percent. Similarly, aggregate consumption in an economy with some monitoring,  $m = 0.05$  no investment in enforcement capacity,  $\phi(e) \rightarrow \infty$ , is 0.72 percent higher if the same allocation can be attained without incurring the monitoring cost. In contrast, eliminating private information frictions in this economy—i.e., setting  $m = 0$ —raises aggregate consumption by 2.57 percent. These efficiency gains are even higher when starting from the economy with some monitoring and enforcement—i.e.,  $m = 0.05$  and  $\phi(e) = 0.5e^2$ . In this case, “rebating” the enforcement and monitoring costs raises aggregate consumption by 0.98 percent, while reducing these costs to zero raises aggregate consumption by 5.49 percent. Lastly, the efficiency gains that arise from changes in the contract structure are higher when the labor supply is less rigid.

## 7 Conclusion

This paper investigates the effects of financial frictions on firm dynamics, aggregate resource allocation, and economic development by proposing and estimating a general equilibrium model in which different forms of financial frictions affect the supply of credit to firms. The optimal dynamic contract at the core of the model bridges the gap between existing cases that arise under either limited enforcement or private information and new cases that arise when both types of financial frictions interact. Estimating the model using data on Colombian manufacturing firms in the 1980s and early 1990s, I find that financial frictions lead to a significant aggregate output loss, and that most of this distortion can be attributed to private information frictions. Reforms that reduce private information frictions lead to economic growth and substantial welfare gains, while reforms that improve contract enforcement without addressing private information frictions do not. There are significant complementarities between the two types of financial reforms, as moral hazard problems tend to be less significant for financial contracting when contracts are more easily enforced. Moreover, a substantial fraction of the financial reform gains stems from changes in the structure of the contract.

Taken together, this analysis suggests two potentially fruitful avenues for further

research. First, the structure of the dynamic contract offers a viable starting point to identify the importance of different forms of financial frictions in data. For example, cross-industry and cross-country heterogeneity in firm and industry dynamics could be exploited to structurally estimate the relative importance of each form of financial friction.<sup>29</sup> Second, the model could be extended to investigate the amplification and propagation of aggregate shocks given different forms of financial frictions. Following the seminal work of [Bernanke and Gertler \[1989\]](#) and [Kiyotaki and Moore \[1997\]](#), a large literature developed to study the amplification and propagation of aggregate shocks under financial frictions—see [Quadrini \[2011\]](#) and [Brunnermeier, Eisenbach, and Sannikov \[2012\]](#) for surveys. A pervasive assumption in this literature is to restrict the horizon of financial arrangements to one period. This assumption places collateral at the center stage of credit allocation, which may amplify and propagate the effect of aggregate shocks under some conditions.<sup>30</sup> When agents can contract over a longer horizon, lending may occur in equilibrium even if entrepreneurs do not have sufficient collateral. In this case, the structure of the financial arrangement could be an important determinant of aggregate fluctuations.

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<sup>29</sup>Preliminary work by [Steri, Schmid, and Nikolov \[2017\]](#) is an important step in this direction.

<sup>30</sup>Recent work by [Hoddenbagh and Dmitriev \[2017\]](#) shows that this type of amplification may not be as large as previously noted if risk-averse, forward-looking entrepreneurs are allowed to sign optimal, state-dependent loan contracts.

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## 8 Figures and tables

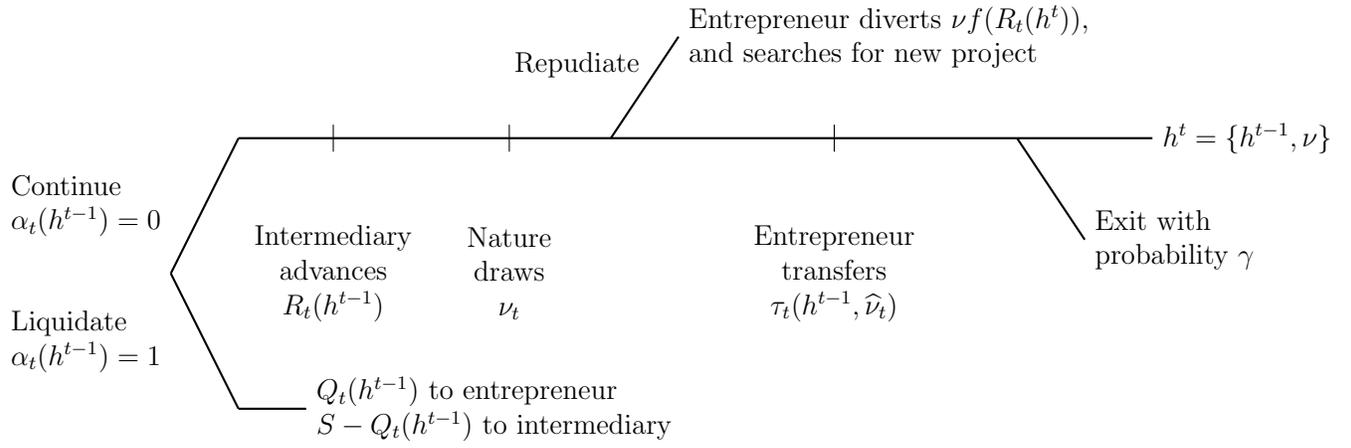


Figure 1: Timing within one period in the benchmark economy.

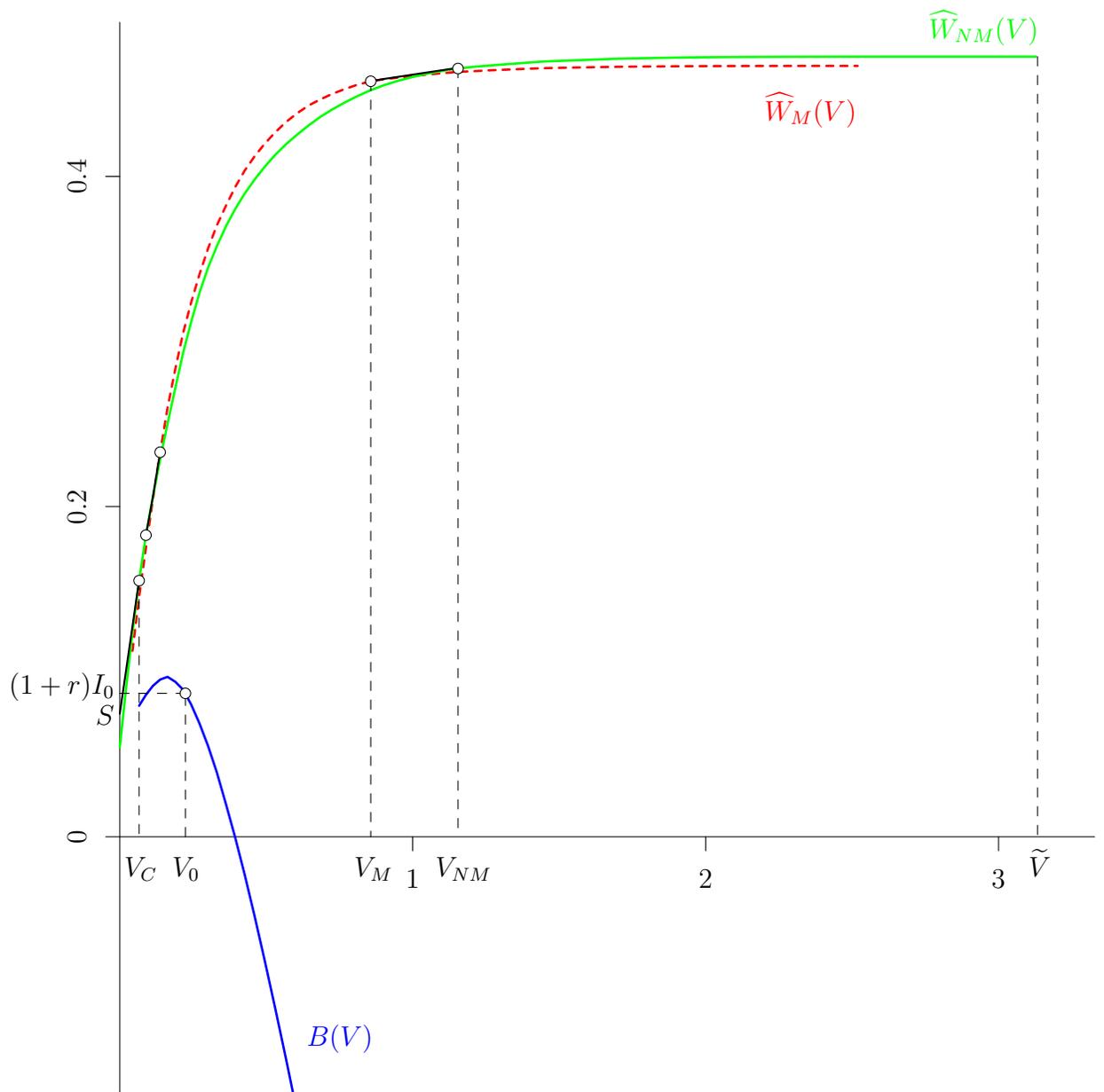


Figure 2: Regions of the contract's optimal value function with endogenous enforcement and monitoring.

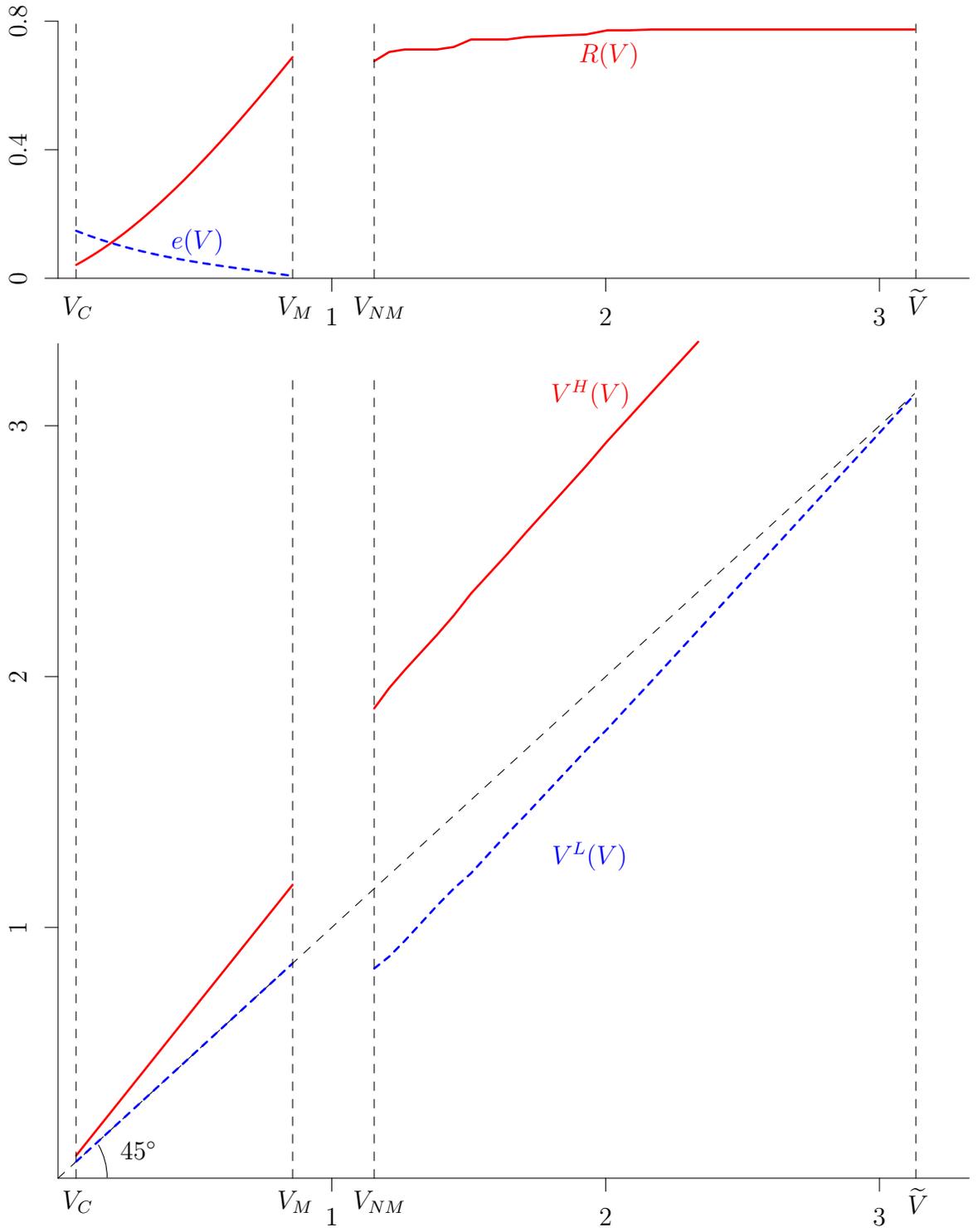


Figure 3: Optimal loan size, enforcement capacity investment, and promised continuation values.

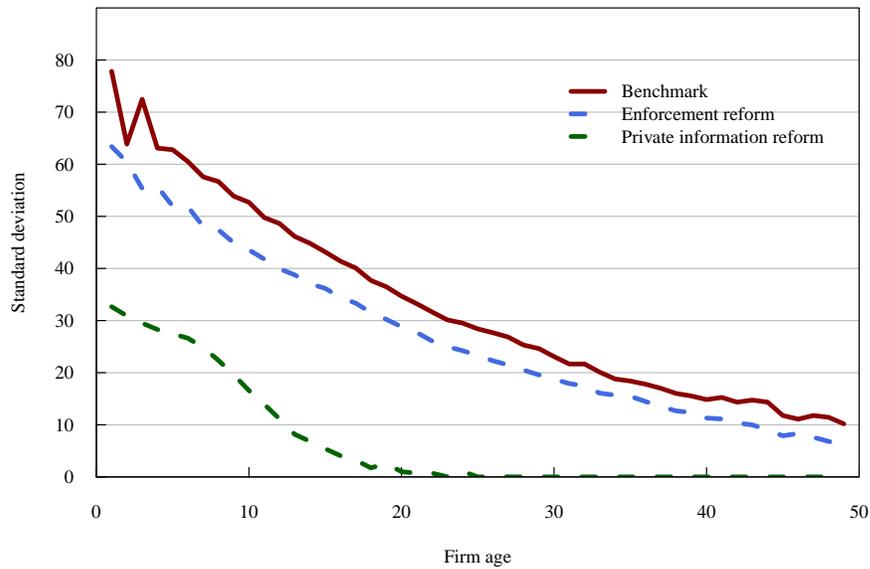
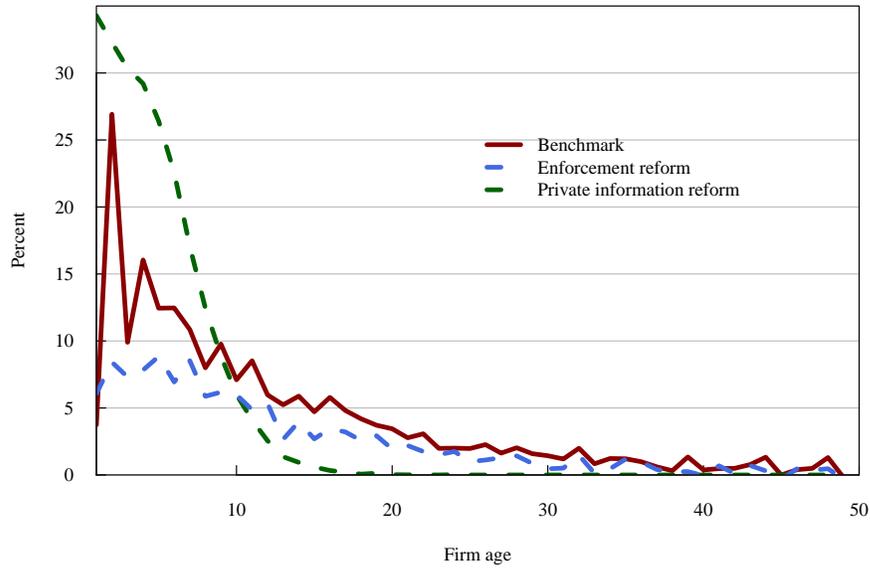


Figure 4: Firm investment growth rate (top) and standard deviation of firm investment growth rate (bottom) conditional on firm age.

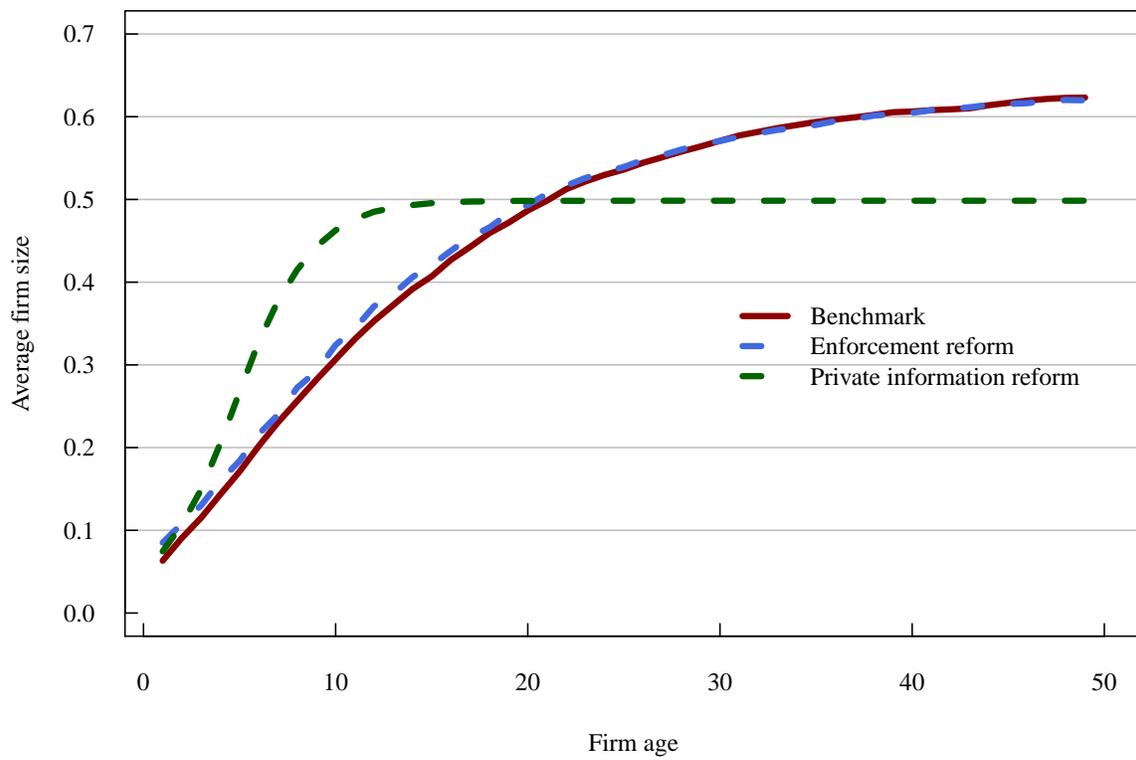


Figure 5: Average firm size in terms of capital conditional on firm age.

(a) Private information reform

(b) Enforcement reform

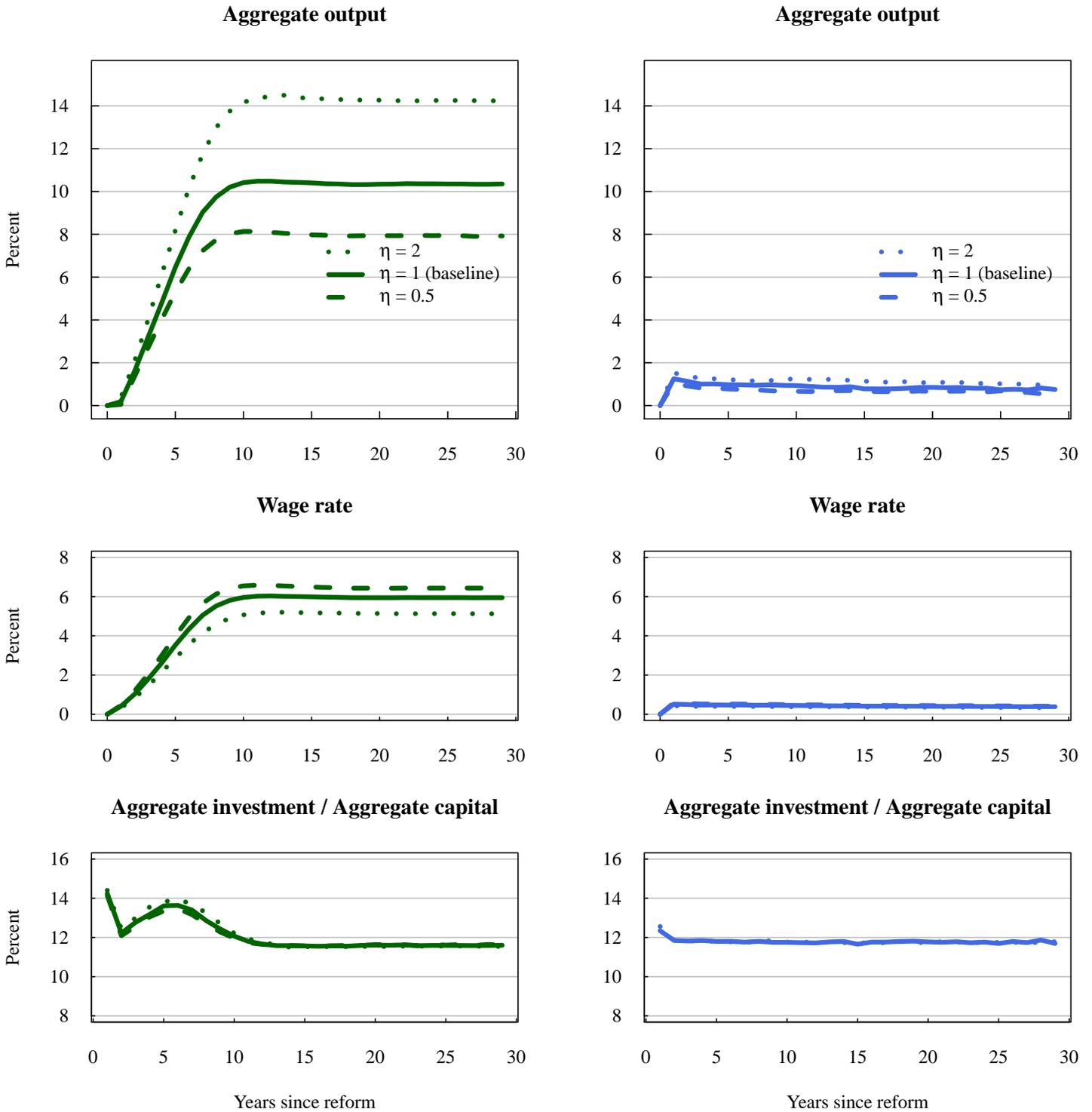


Figure 6: Transition dynamics of financial reforms.

A. Calibration	Parameters	Value	
Interest rate	$r$	0.12	
Worker utility function	$\{A_c, \eta\}$	$\{1.395, 0.5\}$ $\{0.753, 1\}$ $\{0.603, 2\}$	
Firm production function	$\theta$	0.85	
	$\xi$	0.26	
	$z$	1.776	
Contract repudiation cost	$\kappa$	0.45	
B. Panel estimation	Parameters	Estimates	Standard errors
Depreciation rate	$\delta$	0.105	0.025
Cash flow shock	$p$	0.518	0.153
C. SMM estimation	Parameters	Estimates	Standard errors
Fixed set up cost	$I_0$	0.112	0.009
Salvage value	$S$	0.962	0.002
Exogenous exit	$\gamma$	0.059	0.009
	Actual moments	Simulated moments	t-Statistics
Exit rate	0.063	0.062	0.073
Exit rate, small firms	0.12	0.119	0.052
Exit rate, large firms	0.055	0.056	-0.19
Variance of investment, small firms	0.926	0.844	0.225
Average investment (untargeted)	0.32	0.338	-1.946

Table 1: Model parameters.

	Benchmark economy			Frictions eliminated:					
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	Limited enforcement			Private information		
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$
	<i>Percentages relative to first best</i>								
Wage rate	89.69	90.48	91.75	90.00	90.79	92.02	95.28	95.67	96.27
Aggregate output	87.62	84.45	79.68	88.11	85.01	80.41	94.42	93.00	90.89
Aggregate consumption	88.38	85.15	80.28	89.06	85.89	81.18	94.92	93.49	91.34
Hours worked	94.69	90.47	84.17	94.84	90.78	84.66	97.71	95.88	93.10
Liquidation rate (%)	1.04	1.04	1.04	1.27	1.26	1.26	0.00	0.00	0.00
Relative size of new firm (%)	7.31	7.31	7.31	10.59	10.62	10.62	9.87	10.08	10.30
Relative size of the average firm (%)	49.74	49.74	49.74	50.70	50.75	50.76	73.89	74.06	74.33
Fraction of unconstrained firms (%)	28.65	28.65	28.65	29.07	29.10	29.09	40.55	40.55	40.55

Table 2: Steady state allocation and firm dynamics.

	Frictions eliminated:					
	Limited enforcement			Private information		
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$
	<i>Steady-state comparisons</i>					
Workers	0.52	0.69	0.90	9.60	12.04	16.06
Entrepreneurs	-2.63	-2.47	-2.29	-5.81	-3.75	-0.25
Aggregate	-0.38	-0.42	-0.53	5.20	6.52	8.79
	<i>Accounting for transition dynamics</i>					
Workers	0.13	0.29	0.54	5.96	7.41	9.72
Entrepreneurs	-1.12	-0.88	-0.58	4.44	4.60	4.87
Aggregate	-0.23	-0.12	0.04	5.52	6.43	7.56

Table 3: Welfare gains of financial reforms (%).

	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$
<b>A. Welfare gains (%)</b>									
				Enforcement capacity cost					
Monitoring cost	$\phi(e) \rightarrow \infty$			$\phi(e) = \frac{1}{2}e^2$			$\phi(e) = 0$		
$m \rightarrow \infty$	—	—	—	-1.54	-1.84	-2.27	-0.38	-0.42	-0.53
$m = 0.005$	4	5.08	7	4.3	5.44	7.41	—	—	—
$m = 0$	5.2	6.52	8.79	—	—	—	6.31	7.83	10.5
<b>B. Consumption gains (%)</b>									
				Enforcement capacity cost					
Monitoring cost	$\phi(e) \rightarrow \infty$			$\phi(e) = \frac{1}{2}e^2$			$\phi(e) = 0$		
$m \rightarrow \infty$	—	—	—	-0.59	-0.59	-0.59	0.78	0.87	1.12
$m = 0.005$	5.88	7.06	10.63	5.6	7.59	11.29	—	—	—
$m = 0$	7.4	9.79	13.77	—	—	—	10.26	13.5	18.84
<b>C. Efficiency gains (%)</b>									
				Enforcement capacity cost					
Monitoring cost	$\phi(e) \rightarrow \infty$			$\phi(e) = \frac{1}{2}e^2$			$\phi(e) = 0$		
$m \rightarrow \infty$	—	—	—	0.57	0.57	0.57	1.35	1.47	1.71
$m = 0.005$	0.73	0.72	0.69	1.01	0.98	0.93	—	—	—
$m = 0$	1.44	2.57	2.84	—	—	—	4.42	5.49	6.79

Table 4: Financial reform complementarities and sources of efficiency gains

# A Appendix

## A.1 Existence and uniqueness of a recursive stationary equilibrium

In this appendix, I prove existence and uniqueness of a recursive stationary competitive equilibrium in an economy with  $m \rightarrow \infty$  and  $\phi(e) \rightarrow \infty \forall e > 0$ . The argument is similar for the other cases. Given a pair of strictly positive and finite values for the interest rate  $r$  and the wage rate  $w$ , lemma 1 guarantees that perfect competition in financial markets implies that new firms start with a promised equity value  $V_0 < \tilde{V}$ .

### A.1.1 Proof of lemma 1

Pick values for  $\kappa$  and  $I_0$  such that  $\mathcal{T}$  is well-defined on  $[0, \bar{V}_0]$ , where  $\bar{V}_0 < \tilde{V}$ . If  $V_0 = 0$ ,  $\mathcal{O} = (1 - \kappa)\beta V_0 = 0$  and the optimal contract corresponds to the contract studied by Clementi and Hopenhayn [2006]. It is clear that  $\mathcal{T}(0) > 0$  because  $W(V; 0)$  is concave and positive for all  $V \in \mathbf{V}$ . From the discussion in section 3.2, solving problems (12) and (13) with a higher  $V_0$  makes the enforcement constraints bind more because  $\mathcal{O}$  is higher, which reduces the joint surplus  $\widehat{W}(V, V_0)$  for all  $V < \tilde{V}$ . Consequently, a higher  $V_0$  also reduces  $B(V; V_0) = W(V; V_0) - V$  for all  $V < \tilde{V}$ . It follows that  $\mathcal{T}$  is decreasing and therefore has at most one fixed point  $\square$

### A.1.2 Proof of proposition 1

Consider the sequence  $(V_t)_{t \geq 0}$  of entrepreneur promised value corresponding to a single firm indefinitely replaced by a new one of size  $V_0$  upon exit. It is clear that  $(V_t)_{t \geq 0}$  is a sequence of random variables and its evolution depends on the properties of the optimal contracts and the sequence of liquidation, cash flow, and exit shocks. The first step of the proof is to show that, given pair of values for  $r$  and  $w$ ,  $(V_t)_{t \geq 0}$  is a time-homogeneous Markov chain on a general state space, such that

$$V_{t+1} = F(V_t, \epsilon_t), \epsilon \sim \chi, \text{ and } V_0 \sim \psi \tag{36}$$

where  $F : \mathbf{V} \times \mathbf{Z} \rightarrow \mathbf{V}$  is a measurable function,  $\epsilon = (\epsilon_t)_{t \geq 0}$  is a sequence of independent random shocks from a space  $\mathbf{Z}$  with joint distribution  $\chi \in \mathcal{P}(\mathbf{Z})$ , and  $V_0$  is an initial condition from a distribution  $\psi \in \mathcal{P}(\mathbf{V})$ .

To show this, consider the state space  $\mathbf{V}$  of entrepreneur continuation values with a boundedly compact, separable, and metrizable topology  $\mathcal{B}(\mathbf{V})$  and the probability measure space  $(\mathbf{Z}, \mathcal{Z})$  for the shocks. Let  $A$  be any subset of  $\mathcal{B}(\mathbf{V})$ . It follows that for any  $x \in [V_R, \tilde{V})$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \text{ and } V^L(x) \geq V_R \\ (1 - \gamma)(1 - p)\alpha(V^L(x)) & \text{if } A = \{V_0\} \text{ and } V^L(x) < V_R \\ (1 - \gamma)(1 - p)(1 - \alpha(V^L(x))) & \text{if } A = \{V_R\} \text{ and } V^L(x) < V_R \\ (1 - \gamma)(1 - p) & \text{if } A = \{V^L(x)\} \text{ and } V^L(x) \geq V_R \\ (1 - \gamma)p & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

And for  $x = \{\tilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma) & \text{if } A = \{\tilde{V}\} \\ 0 & \text{otherwise} \end{cases} \quad (38)$$

Moreover, for each  $A \in \mathcal{B}(\mathbf{V})$ ,  $P(\cdot, A)$  is a non-negative function on  $\mathcal{B}(\mathbf{V})$ , and for each  $x \in \mathbf{V}$ ,  $P(x, \cdot)$  is a probability measure on  $\mathcal{B}(\mathbf{V})$ .

The next step of the proof is to show that, for a given pair of values for  $r$  and  $w$ ,  $(V_t)_{t \geq 0}$  is globally stable. Let  $\mathcal{G}$  denote the Markov operator associated with the Markov chain  $(V_t)_{t \geq 0}$ , and let  $\mathcal{P}(\mathbf{V})$  denote the collection of equity values distribution generated by  $\mathcal{G}$  for a given an initial distribution  $\psi$ .<sup>31</sup>  $(V_t)_{t \geq 0}$  is globally stable if  $(\psi \mathcal{G}^t)_{t \geq 0} \rightarrow \psi^* \mathcal{G}$  where  $\psi^* \in \mathcal{P}(\mathbf{V})$  is the unique fixed point of  $(\mathcal{P}(\mathbf{V}), \mathcal{G})$ . This occurs

<sup>31</sup>Note that [Stokey, Lucas, and Prescott \[1989, Theorem 12.12\]](#) fail to apply in this case because the stochastic kernel is not monotone on  $[V_R, a]$  where  $a$  is such that  $V^L(a) = V_R$ . For instance, consider the function  $g(x) = x$ . From the above,

$$\begin{aligned} & \int_{[V_C, a]} P(x, dy)y \\ &= (1 - \gamma)\{ (1 - p)[\alpha(V^L(x))V_0 + (1 - \alpha(V^L(x)))V_C] + pV^H(x) \} + \gamma V_0 \\ &= (1 - \gamma)[(1 - p)V_0(1 - V^L(x)/V_C) + (1 - p)V^L(x) + pV^H(x)] + \gamma V_0 \\ &= (1 - \gamma)[(1 - p)V_0\alpha(V^L(x)) + x/\beta] + \gamma V_0, \end{aligned}$$

if  $\mathcal{G}$  is a uniform contraction of modulus  $1 - \vartheta(P)$  on  $\mathcal{P}(\mathbf{V})$  with  $\vartheta(P) > 0$  and where  $\vartheta(P)$  is the Dobrushin coefficient associated with the stochastic kernel  $P(x, dy)$ .<sup>32</sup>

Because a firm exits with fixed, exogenous and independent probability  $\gamma$  each period and is instantaneously replaced by a new one with equity value  $V_0$ , it follows that  $P(x, \{V_0\}) \geq 0 \forall x \in \mathbf{V}$ . Equation (11.15) and exercise (11.2.24) in [Stachurski \[2009\]](#) yield  $\vartheta(p) > \gamma$ . By [Stachurski \[2009, Th. 11.2.21\]](#), it follows that  $\|\psi\mathcal{G} - \psi'\mathcal{G}\|_{TV} \leq (1 - \gamma)\|\psi - \psi'\|_{TV}$  for every pair of distributions  $\psi, \psi'$  in  $\mathcal{P}(\mathbf{V})$ , where  $_{TV}$  indicates the total variation norm.

The rest of the proof consists of establishing that the stationary distribution  $\psi^*$  is continuous in prices  $r$  and  $w$ , which requires checking that the conditions of [LeVan and Stachurski \[2007, proposition 2\]](#) are satisfied. These details are relatively lengthy and are relegated to the online appendix □

### A.1.3 Proof of proposition 2

Proving existence of the recursive stationary competitive equilibrium consists in defining a correspondence from the set of prices to itself using the market clearing conditions and applying the Schauder Fixed-Point [[Stokey et al., 1989, Theorem 17.4](#)]—see the online appendix for details. Uniqueness is difficult to establish with more general preferences than the one used in the text. When the agents' instantaneous utility is linear in consumption, the capital market trivially clears for a given wage rate, which is the only relevant price. Under this assumption, there is no wealth effect, and given a value for  $r$ , an increase in  $w$  yields a lower aggregate demand for labor  $\int n(R(V))d\psi$  and a higher aggregate supply of labor, defined implicitly by  $\varphi'(L) = w$ . It follows that there exists a unique  $w$  that clears the labor market □

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which is generally not increasing. The intuition is that when  $x$  falls below  $a$ , the probability of liquidation  $\alpha(x)$  becomes non-zero in case of a low cash flow shock. However, liquidation sends  $x$  to state  $V_0$ , which can be larger than  $V_R$  and  $V^H(x)$ , so that the lower  $x$  is, the higher  $\mathbf{E}(x'|x)$  is.

<sup>32</sup>The Dobrushin coefficient  $\vartheta(P)$  of a stochastic kernel  $P(x, dy)$  can be defined as  $\vartheta(P) := 1 - \frac{1}{2} \sup_{x,z} \int |P(x, y) - P(z, y)| dy$ .

## A.2 Proof of proposition 3

When  $m \rightarrow \infty$ ,  $\phi(e)$  is low enough, and  $V \geq V_R$ , the first order conditions and the envelope condition (EC) of program (22) and (26) are:

$$R : \quad pf'(R) + (1 - \delta)k'(R) - (1 + r) - \mu_1 f'(R) = 0 \quad (39)$$

$$V^H : \quad pW'(V^H) - p\mu_2 + \mu_1 = 0 \quad (40)$$

$$V^L : \quad (1 - p)W'(V^L) - (1 - p)\mu_2 - \mu_1 + \mu_3 = 0 \quad (41)$$

$$e : \quad -(1 + r)\phi'(e) + \mu_3 = 0 \quad (42)$$

$$\text{EC} : \quad W'(V) - \mu_2 = 0 , \quad (43)$$

where  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are Lagrange multipliers.

When the enforcement constraint conditional on a low cash flow shock does not binds,  $-(1+r)\phi'(e) = 0$  and  $e = 0$  because  $\phi(\cdot)$  is strictly convex. When the enforcement constraint binds,  $\beta V^L(V) = \mathcal{O} - e(V)$ , and equation (27) implies that

$$f'(R) \left[ \phi'(e)(1 + r) + (1 - p) \left( W' \left( \frac{\mathcal{O} - e}{\beta} \right) - W'(V) \right) \right] = \Pi'(R) . \quad (44)$$

Given a level for  $V$ , an increase in  $e(V)$  allows for a higher  $R(V)$  because it increases  $W' \left( \frac{\mathcal{O} - e(V)}{\beta} \right)$  and  $W' \left( \frac{\mathcal{O} - e(V)}{\beta} \right) > W'(V)$ , as  $f(\cdot)$ ,  $\Pi(\cdot)$  and  $W(\cdot)$  are strictly concave on this part of the domain.

It remains to show that  $e(V)$  is nonincreasing in  $V$ . When the enforcement constraint binds and  $e(V) = \mathcal{O} - \beta V^L(V)$ , the result follows if  $V^L(V)$  is nondecreasing in  $V$ . Rearranging once more the above optimal conditions yields,

$$p(1 - p) [W'(V^L(V)) - W'(V^H(V))] + p(1 + r)\phi'(e) = \frac{\Pi'(R(V))}{f'(R(V))} . \quad (45)$$

For the purpose of later contradiction, assume that  $V^L(V)$  is nonincreasing in  $V$ . Consider an increase in  $V$ . The associated decrease in  $V^L(V)$  must be accompanied with an increase in  $V^H(V)$  to maintain incentive compatibility, which implies that  $W'(V^L(V)) - W'(V^H(V))$  is higher because  $W(V)$  is strictly concave on this part of

the domain. In addition,  $\phi'(e)$  is higher because  $\phi(\cdot)$  is strictly convex. Therefore, the term on the right hand side of equation (45) must be higher to maintain optimality. Lemma (2) below derives the condition under which the derivative of the right hand side of equation (44) with respect to  $R$  is negative when there is less than full capital depreciation—i.e.,  $\delta < 1$ .<sup>33</sup>

**Lemma 2.**  $\frac{d}{dR} \frac{\Pi'(R)}{f'(R)} < 0$  iff  $-\frac{d \ln f'(R)}{d \ln k'(R)} > \frac{1-\delta}{r+\delta}$ .

*Proof.* Showing that  $\frac{d}{dR} \frac{\Pi'(R)}{f'(R)} < 0$  is analogous to showing that  $(1 - \delta)k''(R) \cdot f'(R) < [(1 - \delta)k'(R) - (1 + r)] \cdot f''(R)$ , which can be rewritten as  $0 > \frac{d \ln k'(R)}{d \ln f'(R)} > 1 - \frac{1+r}{(1-\delta)k'(R)}$ , because the assumptions on  $\tilde{f}(\cdot)$  implies that  $k'(R) > 0$ ,  $k''(R) \leq 0$ , and  $f''(R) < 0$ . Moreover,  $k'(R) < 1$  whenever  $R < \tilde{R}$  because  $k(R) + w \cdot n(R) = R$ , which implies that  $1 - \frac{1+r}{(1-\delta)k'(R)} > 1 - \frac{1+r}{(1-\delta)}$   $\square$

Assuming that  $\delta$  and  $r$  are not too small so that the condition of lemma (2) holds, Equation (45) requires that a higher  $V$  be associated with a lower  $R$ . However, this is a contradiction, as an increase in  $V^H(V) - V^L(V)$  must be associated with an increase in  $R$  to maintain incentive compatibility  $\square$

### A.3 Proof of proposition 4

When  $m = 0$ ,  $\phi(e)$  is low enough, and  $V \geq V_C$ , the first-order conditions and the envelope condition of program (22) and (26) are

$$R : \quad p f'(R) + (1 - \delta)k'(R) - (1 + r) - \mu_1 f'(R) = 0 \quad (46)$$

$$V^H : \quad p W'(V^H) - p \mu_2 + \mu_1 = 0 \quad (47)$$

$$V^L : \quad (1 - p)W'(V^L) - (1 - p)\mu_2 + \mu_3 = 0 \quad (48)$$

$$e : \quad -(1 + r)\phi'(e) + \mu_1 + \mu_3 = 0 \quad (49)$$

$$\text{EC} : \quad W'(V) - \mu_2 = 0, \quad (50)$$

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<sup>33</sup>Note that  $\frac{d}{dR} \frac{\Pi'(R)}{f'(R)} < 0$  when  $\delta = 1$ , as  $\frac{\Pi'(R)}{f'(R)} = 1 - \frac{(1+r)}{f'(R)}$ . Thus, the condition in lemma (2) requires that capital be relatively costly whenever the intermediary can recoup  $(1 - \delta)k$  with certainty by borrowing  $R$  at rate  $r$  and investing it in the risky project.

where  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are Lagrange multipliers.

The enforcement constraint conditional on a low cash flow realization never binds so that  $\mu_3 = 0$  for all  $V$ . To see this effect, assume for the purpose of contradiction that it binds and  $\mu_3 > 0$  so that  $\beta V^L(V) = \mathcal{O} - e(V)$ . Substitution into the promise keeping constraint yields  $V \geq \beta(pV^H(V) + (1-p)\beta(\mathcal{O} - e(V)))$ . This implies that  $V \geq \mathcal{O} - e(V)$ , which violates the enforcement constraint conditional on a high cash flow shock realization. Given that  $\mu_3 = 0$ ,  $W'(V^L(V)) = \mu_2$  and the envelope condition implies that  $V^L(V) = V$ . In this case, promise-keeping implies that  $V^H(V) = V(1 - \beta(1 - p))/(\beta p)$  and rearranging the first order condition yields equation (30). It follows that  $e = 0$  when  $R(V) = \tilde{R}$ , as  $pf'(\tilde{R}) + (1 - \delta)k'(\tilde{R}) = (1 + r)$ , which holds for all  $V \geq V_u$ , where  $V_u$  is the smallest value such that  $R(V_u) = \tilde{R}$ . When  $V < V_u$ ,  $e > 0$  and Equation (30) implies that a higher  $e$  is associated with a higher  $R$ . Lastly, the first order conditions and the envelope condition yield  $(1 + r)\phi'(e) = W'(V) - ((1 - p)W'(V^L(V)) + pW'(V^H(V)))$ , which simplifies to  $(1 + r)\phi'(e) = pW'(V) + pW'(V(1 - \beta(1 - p))/(\beta p))$  and implies that  $e$  is strictly decreasing in  $V$ , as  $(1 - \beta(1 - p))/(\beta p) > 1$  with  $\beta < 1$   $\square$

#### A.4 Proof of proposition 5

Consider an arbitrarily small  $m > 0$ . It is clear that  $\widehat{W}_M(\tilde{V}) < \widehat{W}_{NM}(\tilde{V})$  and  $\beta S - m = \widehat{W}_M(0) < \widehat{W}_{NM}(0) = \beta S$ . From the above discussion, there exists a value  $V_u < \tilde{V}$  for the entrepreneur such that  $\widehat{W}_M(V_u) = \widehat{W}_M(\tilde{V})$ . The result follows because  $\widehat{W}_{NM}(\tilde{V})$  is strictly concave on  $[0, \tilde{V}]$   $\square$