Aggregate Consequences of Dynamic Credit Relationships*

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Abstract

I investigate the aggregate consequences of canonical financial frictions in the supply of credit to firms: private information and limited enforcement. I propose a general equilibrium model in which entrepreneurs finance their firm through a long-term contract with a financial intermediary. The contract is constrained efficient because firm cash flow is costly to monitor and borrowers that repudiate cannot be excluded from capital markets. By investing in enforcement capacity, an intermediary can delay debt repayment and maintain incentive compatibility. Reforms that seek to decrease the cost of monitoring or enforcing contracts, or both, affect firm dynamics and can generate complementaries. Estimating the model with data on Colombian manufacturing firms in the 1980s and 1990s, I find that financial frictions are responsible for a significant aggregate output loss. Most of this distortion can be attributed to private information. Reforms that only reduce private information create significant economic growth and welfare gains, while those that only improve enforcement do not. There are significant complementarities between different types of reforms, as moral hazard is less significant when contracts are more enforceable.

JEL Codes: E22, D82, G32, L14, O11, O54

Keywords: private information; limited enforcement; dynamic financial contracting; financial development; firm dynamics; Colombia

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*This version: November 22, 2016. This paper has greatly benefited from the comments and suggestions of two anonymous referees, Vincenzo Quadrini (the editor), Peter Rupert, Finn Kydland, Espen Henriksen, Marek Kapicka, Till Gross, Thomas Cooley, John Stachurski, Borghan Narajabad, Giovanni Favara, Cyril Monnet, Víctor Ríos-Rull, Nathan Foley-Fisher and seminar participants at the Annual Meeting of the Society for Economic Dynamics (Toulouse), LAEF-Macroeconomics and Business Cycle, Midwest Macro, Georgia State University, Florida State University, University of New South Wales, University du Quebec à Montréal, UC Santa Barbara, the Federal Reserve Board and Fundação Getulio Vargas. I would also like to thank the Colombian Departamento Administrativo Nacional de Estadística (DANE) for providing access to the data used in this paper and for their generous hospitality extended while visiting their offices in Bogota, Colombia. The views expressed in this paper do not necessarily reflect the views of the Board of Governors of the Federal Reserve System, or its staff.

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1 Introduction

Which financial frictions matter for aggregate resource allocation and for economic development and growth? A large microeconomic literature on firm and industry dynamics has studied financial frictions that arise when information about firms is expensive to acquire and when financial contracts are difficult to enforce. The cost of acquiring information and limits to contract enforceability are incentives that result in different types of financial contracts [Levine, 2005]. These differences shape the supply of credit to firms, and, in the aggregate, the efficiency at which the economy transforms savings into investment. Although there is broad consensus that financing frictions make it costly for firms to raise external finance (see Hubbard [1998] and Stein [2003] for surveys), there remains considerable debate surrounding the importance of financial frictions for the determination of macroeconomic outcomes.[1]

The main contribution of this paper is to propose and estimate a general equilibrium model in which different forms of financial frictions affect financial contracting and the supply of credit to firms. Recent developments in the theory on dynamic financial contracting have shown that private information and limited enforcement frictions can individually account for some of the empirical regularities on firm dynamics.[2] Although many properties of this class of financial contracts are well understood, their implications for the determination of macroeconomic outcomes remain largely unexplored.[3]

In the model, entrepreneurs with uncertain lifetimes can operate a long-lived firm. Starting a firm requires incurring a partially sunk fixed cost and working capital each period. New entrepreneurs do not have enough resources to finance their firms and

[1] For example, Gilchrist, Sim, and Zakrajsek [2013] and Midrigan and Xu [2014] argue that the misallocation due to financial frictions is much lower than previous estimates.

[2] Empirical studies of firms have shown that smaller and younger firms pay fewer dividends, take on more debt, and experience more rapid but also more volatile growth, and that small and young firms' investment is more sensitive to cash flows (Cooley and Quadrini [2001], Cabral and Mata [2003], Oliveira and Fortunato [2006], Fagiolo and Luzzi [2006], and Lu and Wang [2010]). The seminal contributions on dynamic financial contracting to account for firm dynamics include Albuquerque and Hopenhayn [2004], Quadrini [2004] and Clementi and Hopenhayn [2006]. See also Hopenhayn and Werning [2008], Li [2013], Kovrijnykh [2013], and Popov [2014].

[3] The seminal work in this area is Cooley, Marimon, and Quadrini [2004] and Smith and Wang [2006]. More recent work includes Li [2010] on employment flows, Dyrda [2014] on the effect of aggregate uncertainty shocks, Gross and Verani [2011] and Dovis and Brooks [2014] on the context of international trade, and Veracierto [2014] on the context of consumers decisions.

may obtain external financing by entering into a long-term contract with a financial intermediary. Once started, firms generate a random cash flow that is a function of the working capital input. The financial contract is constrained efficient because firms operate under limited liability, revenues are costly to monitor, and entrepreneurs repudiating their debt are not excluded from capital markets. A financial intermediary may invest in enforcement capacity to reduce the value of the entrepreneur's outside option, but doing so is costly.

Financial development in this model is a combination of lower costs of monitoring firms and enforcing contracts. A less financially developed economy is subject to more severe private information and limited enforcement frictions because the costs of monitoring firms and enforcing contracts are higher. Reforms to increase financial development in the model aim to reduce either one or both of these costs, which expands the set of feasible contracts and can thus have significant effects on firm dynamics.

The financial arrangement at the core of the general equilibrium model adds to the literature on dynamic contracting by considering the role of multi-period contracting, limited enforcement, and imperfect capital market exclusion in an environment with private information.[4] The optimal dynamic contract analyzed in this paper nests a number of seminal cases that arise under either private information [Quadrini, 2004, Clementi and Hopenhayn, 2006] or limited enforcement [Albuquerque and Hopenhayn, 2004, Popov, 2014]. Consistent with the predictions of these models, younger and smaller firms tend to grow disproportionately faster than older and larger firms. In addition, new properties arise when private information and limited enforcement frictions interact with each other and when the intermediary can mitigate their adverse effects by investing in some technology. For example, investing in enforcement capacity under private information has two effects on the optimal contract. As in Popov [2014], it provides insurance against debt repudiation by decreasing the value of the entrepreneur's outside option. In addition, a lower value of debt repudiation allows the intermediary to delay debt repayment while maintaining incentive compatibility at smaller firms,

[4]See previous work by Atkeson [1991], Khan [2001], Castro, Clementi, and Macdonald [2009], Buera, Kaboski, and Shin [2011], Greenwood, Sanchez, and Wang [2013] and Arellano, Bai, and Zhang [2012].

which increases the supply of credit to these firms. This mechanism is central to the complementarities that arise in the aggregate with reforms that decrease both the cost of monitoring firms and of enforcing contracts.

I investigate the aggregate effects of financial frictions by estimating a benchmark model economy using data from Colombia in the 1980s and 1990s and by conducting a series of counterfactual reform experiments. Colombia in this period provides an ideal benchmark to analyze the effect of different types of financial reforms, as a number of institutional and legal features prevented efficient firm monitoring and contract enforcement. An innovation in this paper is to use a simulated method of moment estimation (SMM) procedure more widely used in the dynamic corporate finance literature to structurally estimate the contract parameters using firm-level data.[5]

The quantitative analysis yields four main results. First, financial frictions lead to a substantial misallocation of resources. Aggregate output in the benchmark model economy is about 17 percent lower than its potential first-best level. Although limited enforcement frictions can by themselves cause an 8 percent aggregate output loss, their relative effects in the presence of private information are small—that is, private information absent any enforcement frictions leads to an output loss that is nearly 90 percent of the output loss in the benchmark economy. Second, financial development can lead to significant economic growth. A reform that eliminates private information frictions leads to roughly 11 consecutive years of economic growth at an average growth rate of 1 percent per year before the economy reaches its new steady state. This effect is in sharp contrast to a reform that only eliminates enforcement frictions. In this case, aggregate output immediately rises by about 1.5 percentage points and grows by an additional half of a percent during the next 30 years. Third, while both type of reforms yield aggregate welfare gains, eliminating private information frictions yields welfare gains that are about 14 times larger than those associated with a reform that eliminates limited enforcement frictions. Fourth, there are significant complementarities between different types of financial reforms. The effect of a reform that reduces both the enforcement and private information frictions yields aggregate welfare gains that are on average between

[5] See Strebulaev and Whited [2012] for a survey of the recent dynamic corporate finance literature.

1 and 2 percentage points higher than the combined aggregate welfare gains associated with the implementation of individual reforms of the same magnitude.

One of the main mechanisms driving these aggregate results is that, in the presence of private information frictions, limited enforcement frictions only affect the smallest and most inefficient firms in the economy. Consequently, a reform that improves contract enforcement relaxes the financial constraints on these small firms and only has a limited effect on aggregate resource allocation and welfare. When private information frictions are reduced, the economy grows as a large mass of small incumbents slowly converges to the new stationary distribution in which firms are, on average, larger, demand more labor, and pay a higher wage. In addition, complementarities between the two types of reforms arise because young firms are, on average, larger when enforcement capacity is high. Because young entrepreneurs can build a greater stake in their firm more rapidly with high enforcement capacity, the moral hazard problem becomes less significant.

Taken together, these results provide new answers to a number of questions regarding the aggregate effects of financial frictions. Regardless of the form of financial frictions, financial development leads to firms that, on average, are larger and grow slower. Thus, although the cross-country heterogeneity in firm growth noted by Arellano et al. [2012] could be driven by cross-country differences in contract enforcement, the analysis suggests that it could also be driven by cross-country differences in the efficacy of the monitoring technology as emphasized in Greenwood, Sanchez, and Wang [2010] and Greenwood et al. [2013]. Moreover, the quantitative results are consistent with Midrigan and Xu [2014] insofar as the misallocation associated with limited enforcement conditional on the effect of private information is small, as firms can rapidly grow out of the borrowing constraints implied by limited enforcement. However, the effect of private formation on misallocation is several orders of magnitude greater than that of limited enforcement, as the incentive compatibility constraint continues to bind for larger firms. Lastly, the analysis contributes a nuanced answer to the seminal question posed by Goldsmith [1969] as to whether financial development causes economic growth. In the model, financial development causes significant economic growth if it is associated with a reduction of private information frictions. However, the effects of financial development on economic growth are more modest if a reform that improve contract enforcement fails to address private information frictions.

The rest of the paper proceeds as follows. Sections 2 and 3 present the environment and the financial contract, respectively. Section 4 defines the general equilibrium. Section 5 discusses the properties of the optimal contract and qualitatively analyzes the effect of different types of financial reforms on the optimal contract. Section 6 discusses the parameterization of the benchmark model economy and the main results from the quantitative analysis; and Section 7 concludes. Proofs of propositions and details of the numerical strategies are relegated to the Appendix.

2 Model

Time is discrete and infinite, and each period is indexed by t. The economy is populated by continuums of entrepreneurs and workers. An agent's career path is an endowment and cannot be altered. Workers have an infinite lifetime, while entrepreneurs have an uncertain lifetime. Long-lived firms managed by entrepreneurs use labor supplied by workers and capital to produce a numéraire good used for consumption and investment.

2.1 Workers

There is a mass λ of workers endowed with one unit of time each period. Workers allocate their time between work and leisure, and they discount the future at rate $(1+r)^{-1}$, where r is the real interest rate.[6] Workers choose consumption c_t and labor hours $h_t \in [0,1]$ to maximize the value of their lifetime expected utility[7]

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [c_t - \varphi(h_t)] \ , \tag{1}$$

[6] I am anticipating that the real interest rate r is constant in equilibrium.

[7] The assumptions on the workers are borrowed from Cooley et al. [2004].

subject to the period budget constraint

$$c_t + d_{t+1} \leq (1+r)d_t + w_t h_t \ , \tag{2}$$

and $d_{t+1} \geq -\epsilon$, where w_t is the wage rate, d_t is the asset position at time t and ϵ is a natural borrowing limit. The function $\varphi(\cdot)$ is strictly concave and satisfies $\varphi(0) = 0$.

2.2 Entrepreneurs

There is a unit mass of entrepreneurs born without wealth that survive into the next period with fixed probability $(1-\gamma)$ and are replaced by new ones upon death. Entrepreneurs are risk-neutral and discount the future at rate $(1-\gamma)/(1+r)$. Entrepreneurs choose their consumption to maximize the value of their lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1-\gamma}{1+r} \right)^t c_t \ . \tag{3}$$

Entrepreneurs do not participate in the labor market and may start a long-lived firm to generate consumption.

2.3 Technology and firms

Long-lived firms can be built according to a blueprint technology and operated by entrepreneurs. There is one type of blueprint in the economy, and every entrepreneur not already operating a firm has access to one. The blueprint is a plan to set up a firm with decreasing returns to scale technology $f(k,n)$, where k and n are capital and labor inputs, respectively. The production function is strictly increasing in labor and capital, strictly concave, and satisfies $f(k,0) = f(0,n) = 0$. Starting a firm requires paying an initial fixed cost I_0, which is partially sunk as firms can always be liquidated with scrap value $S < I_0$.

Once started, a firm requires working capital R each period that is used to rent capital k and to hire labor n at wage w before production can take place. The capital used in production depreciates at rate $\delta \in (0,1)$. Firm cash flow is subject to a sequence of independent and identically distributed idiosyncratic shocks $(\nu_t)_{t\geq 0}$, where $P(\nu = 1) = 1 - P(\nu = 0) = p$. Expected gross revenue in any period is given by

$$pf(R) + (1-\delta)k(R) := \max_{k,n} pf(k,n) + (1-\delta)k \quad \text{s.t. } k + wn \leq R \ , \tag{4}$$

which is the sum of the expected cash flow $pf(R)$ generated using working capital R plus the amount of undepreciated capital $(1-\delta)k(R)$ that is left after production takes place.

The assumptions about the firm technology imply there exists a unique level of working capital $\tilde{R}$ that maximizes expected profit, such that

$$\tilde{R} := \underset{R}{\operatorname{argmax}}\, pf(R) + (1-\delta)k(R) - (1+r)R \tag{5}$$

for a given interest rate r and wage rate w. The assumption of diminishing returns to scale with fixed start-up costs implies that it can be optimal to organize production around large firms. A firm is terminated upon the death of its entrepreneur, which occurs with probability γ and is analogous to receiving a permanent zero-productivity shock.[8] It is profitable to start a firm only if the expected discounted lifetime profit of a firm operating at full scale is greater than the cost of starting the firm—that is,

$$\frac{pf(\tilde{R}) + (1-\delta)k(\tilde{R}) - (1+r)\tilde{R}}{1-\beta} > (1+r)I_0 \ , \tag{6}$$

where $\beta = (1-\gamma)/(1+r)$. Lastly, entrepreneurs can only manage one firm, and starting a new firm requires an entrepreneur to abandon her current firm.

2.4 Financial intermediation and financial frictions

Financial intermediaries are institutions that arise to provide workers and entrepreneurs a technology to save and borrow. Financial intermediaries offer long-term contracts to

[8] This assumption captures other sources of firm exit not modeled explicitly and allows me to pin down the steady-state distribution of firms without keeping track of individual firms—see, for instance, Cooley and Quadrini [2001], Cooley et al. [2004], and Smith and Wang [2006].

entrepreneurs to finance their firms in exchange for payments. Financial intermediaries can fully commit to these contracts, but entrepreneurs operating their firms under limited liability may repudiate their debt. In case of repudiation, an entrepreneur absconds with the period cash flow, loses the firm, and may enter into a new contract to start a new firm. The diverted cash flow cannot be invested in a new project.

The value of repudiating a contract is the sum of the period random cash flow $\nu \cdot f(R)$, which is non-decreasing in working capital R, and the external value of setting up a new project net of any repudiation costs denoted by $\mathcal{O}$, which is determined in general equilibrium. In addition to limited contract enforcement, entrepreneurs have private information about ν and may misreport the period cash flow to increase their consumption. The combination of limited enforcement and private information under limited liability leads to financial frictions that affect contracting with entrepreneurs.

Payments from the entrepreneurs to the financial intermediary are used to repay outstanding debt and to accumulate deposits. Financial intermediaries use deposits from workers and entrepreneurs to finance their portfolio of firms. One unit of workers' deposits at the beginning of a period pays $1+r$ units at the end of the period. Because the deposits accumulated by deceased entrepreneurs are available to finance firms in an intermediary's portfolio, the rate of return on entrepreneurs' deposits is $(1+r)/(1-\gamma)$. Perfect competition and free entry in the financial intermediation industry imply that any coalition of entrepreneurs can establish a financial intermediary. In equilibrium, who owns a financial intermediary is irrelevant because of zero profit. Consequently, I focus on a single representative financial intermediary in the remainder of the paper.

3 Optimal financial contracting in the benchmark model economy

In the benchmark economy, I assume that a financial intermediary cannot monitor firm cash flows and cannot invest in enforcement capacity to decrease $\mathcal{O}$. An interpretation of this benchmark is an economy in which the costs of investing in firm monitoring and enforcement capacity are so high that they are never incurred by the intermediary. These high costs could reflect, for example, the lack of institutional and legal infrastructures necessary to support consistent corporate financial reporting and recourse lending and to exclude defaulting entrepreneurs from capital markets. In Section 5, I study the effects of reforms that decrease these costs on financial contracting and the supply of credit to firms.

The financial intermediary has the same discount rate as entrepreneurs and offers them a state-contingent financial contract that induces truthful reporting and debt repayment each period. Denote the reporting strategy of an entrepreneur by the sequence of reports $\underline{\hat{\nu}} = \{\hat{\nu}_t(\nu^t)\}_{t\geq 0}$, where $\nu^t = (\nu_1, \ldots, \nu_t)$ is the true history of the revenue shocks experienced by the entrepreneur. The history of reports is denoted by $h^t = (\hat{\nu}_1, \ldots, \hat{\nu}_t)$. A financial contract $\sigma = \{\ell_t(h^{t-1}), \hat{\nu}_t), Q_t(h^{t-1}), R_t(h^{t-1}), \tau(h^{t-1}, \hat{\nu}_t)\}$ specifies a liquidation rule $\ell_t(h^{t-1}) \in \{0, 1\}$, transfer $Q_t(h^{t-1}) \in \mathbb{R}_+$ from the intermediary to the entrepreneur in the event of liquidation, period working capital $R_t(h^{t-1}) \in \mathbb{R}_+$, and transfers $\tau(h^{t-1}, \hat{\nu}_t) \in \mathbb{R}$ between the entrepreneur and the financial intermediary conditional on the ex-post cash flow report $\hat{\nu}_t$.

The timing within a period is as follows. At the start of the period, the intermediary decides whether the firm should be liquidated. If the firm is liquidated, the intermediary transfers $Q_t(h^{t-1})$ to the entrepreneur and recoups $S - Q_t(h^{t-1})$. If the firm is not liquidated, a working capital $R_t(h^{t-1})$ is loaned to the entrepreneur to hire labor and rent capital before production takes place. After the period cash flow is realized, the entrepreneur decides whether to continue with the contract or to repudiate the contract. If she does not repudiate the contract, the entrepreneur makes a transfer $\tau(h^{t-1}, \hat{\nu}_t)$ to the intermediary based on her report $\hat{\nu}_t$. At the end of the period, the firm either permanently exits with probability γ or remains in operation. Figure 1 summarizes the sequence of events within one period.

After history h^{t-1}, the pair of contract and report strategy $(\sigma, \underline{\hat{\nu}})$ implies an expected discounted cash flow $V_t(\sigma, \underline{\hat{\nu}}, h^{t-1})$ and $B_t(\sigma, \underline{\hat{\nu}}, h^{t-1})$ for the entrepreneur and the financial intermediary, respectively. Following Green [1988], Spear and Srivastava [1987], and others, the contract can be solved recursively using V_t and V_{t+1} as the continuation values awarded to the entrepreneur contingent on her reports. Following Quadrini [2004], Albuquerque and Hopenhayn [2004], and Clementi and Hopenhayn [2006], a feasible and incentive-compatible contract is optimal if it maximizes $B_t(V_t)$ for every feasible V_t. This maximization problem is analogous to maximizing the value of the joint surplus $W_t(V_t) = B_t(V_t) + V_t$, which can be interpreted as the value of the firm by interpreting V_t and $B_t(V_t)$ as debt and equity, respectively.

Let V^L and V^H denote the promised continuation values awarded to an entrepreneur reporting low and high cash flow, respectively. Because the optimal contract induces truth-telling, I do not distinguish between the report $\hat{\nu}(\nu)$ and the actual realization ν unless otherwise necessary. The optimal contract must satisfy five constraints. First, feasibility requires that the transfers from the entrepreneur to the intermediary should not exceed the period cash flow, as entrepreneurs do not have access to an alternative saving technology. Referring to τ as the transfer to the intermediary after a high report and assuming that the firm always efficiently allocates the working capital it has at its disposal, the feasibility constraint simplifies to

$$\tau \leq f(R) \ , \tag{7}$$

where $f(R)$ is the cash flow generated by a firm with access to working capital R conditional on a high shock. Second, the entrepreneur's value at the beginning of the period, V, should be equal to the expected period cash flow net of repayment plus the discounted expected promised continuation value:

$$V = p(f(R) - \tau) + \beta(pV^H + (1-p)\beta V^L) \ . \tag{8}$$

Third, incentive compatibility requires that following a high report, transfers from the entrepreneur plus the discounted high promised continuation value, V^H, be no less than the value of diverting and immediately consuming the period cash flow and receiving a low continuation value V^L next period:

$$f(R) - \tau + \beta V^H \geq f(R) + \beta V^L \ . \tag{9}$$

Fourth, debt repayment requires that the value derived from repaying the debt is greater than the value of diverting the period cash flow and pursuing another opportunity denoted by $\mathcal{O}$. Given that the incentive compatibility constraint holds, debt repayment requires that

$$\beta V^L \geq \mathcal{O} \ . \tag{10}$$

The value $\mathcal{O}$ is determined in general equilibrium but is treated as a constant for now. Fifth, firms operate under limited liability that requires that

$$V^L \geq 0 \text{ and } V^H \geq 0 \ . \tag{11}$$

Conditional on the entrepreneur being alive, the intermediary can recover S by liquidating the firm at the start of a period. The highest joint surplus can be achieved by randomizing the liquidation decision such that

$$\begin{aligned} W(V) = \max_{\alpha \in [0,1], Q, V_C} \quad & \alpha S + (1-\alpha)\widehat{W}(V_C) \\ & \text{s.t. } \alpha Q + (1-\alpha) V_C \geq V \ , \end{aligned} \tag{12}$$

where $\alpha(V)$ is the probability that the firm is liquidated. If the firm is not liquidated with probability $1 - \alpha(V)$, the entrepreneur receives continuation value V_C, and the value of the joint surplus conditional on the firm avoiding liquidation is given by

$$\begin{aligned} \widehat{W}(V) = \max_{\tau, R, V^L, V^H} & \, pf(R) + (1-\delta)k(R) - (1+r)R + \beta(pW(V^H) + (1-p)W(V^L)) \\ & \text{s.t. } (8), (9), (10), (7), \text{ and } (11) \ . \end{aligned} \tag{13}$$

4 General equilibrium

Perfect competition in the financial intermediation industry requires that the representative financial intermediary breaks even on a new contract. This break-even condition implies that the initial value to the entrepreneur, V_0, yields the greatest feasible value such that the financial intermediary makes zero profit:

$$V_0 = \sup_V \{B(V) - (1+r)I_0 = 0\} \ . \tag{14}$$

The entrepreneur's external value of searching for a new project net of repudiation cost $\mathcal{O}$ is given by

$$\mathcal{O}(V_0) = (1-\kappa)\beta V_0 \ , \tag{15}$$

where $\kappa\beta V_0$ is the cost of repudiating the contract and $\kappa \in (0,1)$ is a fixed parameter later set to ensure the contract is feasible. As a result, the initial firm value V_0 enters the optimal contracting problem via $\mathcal{O}$ in the enforcement constraints. As in Cooley et al. [2004], solving for the optimal contract requires solving for the fixed point of a mapping T that maps a set of functions $\mathcal{O}$ into itself, so that

$$\mathcal{O}^{j+1} = T(\mathcal{O}^j) \ . \tag{16}$$

Given the function $\mathcal{O}^j$ that determines that value of repudiating the contract in the next period, T returns the value $\mathcal{O}^{j+1}$ of repudiating the contract today.

The initial firm value V_0 determines the working capital $R(V_0)$ and total debt $R(V_0)+I_0$ of a new firm. New and constrained entrepreneurs can increase their stake in their firm by making positive transfers to the intermediary to repay their debt and accumulate deposits. I discuss the dynamics of V and $R(V)$ in the next section. Let $\mathbf{M}$ be the state space for entrepreneurs' value, so that $V \in \mathbf{M}$. Let $\mathcal{M}(V)$ be the Borel $\sigma-$algebra generated by $\mathbf{M}$, and μ the measure of firms defined over $\mathcal{M}$. Proposition 1 states that for a given set of prices r and w the distribution of firms converges to a stationary distribution in a finite number of periods.

Proposition 1 *For given prices r and w, there exist a stationary distribution of firms that is ergodic.*

At any point in time, the intermediary holds a portfolio of contracts indexed by V so that the aggregate net deposit by entrepreneurs is

$$D_e = -\int B(V) d\mu \ , \tag{17}$$

which could be positive or negative depending on the shape of the distribution of firms μ implied by the financial contract. Recalling that the intermediary owns the firm's capital stock, the value of an unconstrained entrepreneur's contract to the intermediary is $B(\tilde{V}) = [(1-\delta)k(\tilde{R}) - (1+r)\tilde{R}]/(1-\beta)$, which is negative. A positive D_e implies the intermediary can use deposits from the entrepreneurs managing larger firms to finance smaller firms. A negative D_e implies that the intermediary must raise additional deposits D_w from workers to finance the portfolio of firms. It follows that the capital market clears when the net deposits from workers D_w and entrepreneurs D_e are just enough to finance the aggregate working capital and the set-up cost of new firms:

$$D_e + D_w = \int R(V) d\mu + \hat{\gamma}(I_0 - S) + \gamma I_0 \ , \tag{18}$$

where $\hat{\gamma}$ is the fraction of firms that are liquidated each period. The labor market clears when labor demand from firms is equal to labor supplied by the workers, such that

$$H = \int n(V) d\mu \ , \tag{19}$$

where the wage rate is determined competitively by $\varphi'(H) = w$. Given that the labor and the capital market clears, the goods market also clears and aggregate output is divided between worker and entrepreneur consumption and aggregate investment.[9] The definition of a general equilibrium in the economy is as follows.

Definition 1 *A general equilibrium consists of labor supply and consumption function $h(d)$ and $c(d)$ for workers, a contract $\{R(V), \tau(V), V^L(V), V^H(V), \alpha(V), Q, V_C\}$, an initial firm value V_0, prices w and r and a mapping T such that*

1. *the labor and consumption functions maximize workers' utility;*

[9] See the Appendix for more details.

2. *the financial contract maximizes the value of the firm;*

3. V_0 *is such that the intermediary breaks even on new contracts;*

4. $\mathcal{O}$ *is the fixed point of* T*;*

5. w *and* r *clear the labor and capital market.*

Proposition 2 below establishes the existence and uniqueness of a stationary equilibrium by using the Markov properties of the model and the results on the parametric continuity of stationary distributions from LeVan and Stachurski [2007].

Proposition 2 *There exists a unique stationary equilibrium.*

5 Properties of the optimal contract

Assume that the economy has reached its steady state so that prices and the outside option value $\mathcal{O}$ are constant. After the contract is signed, the intermediary finances the set-up cost I_0 and the initial working capital R_0. Because entrepreneurs with zero net worth have an incentive to misreport the cash flow and repudiate their debt, the initial loan R_0 is less than the working capital needed by the firm to operate at its efficient scale, $\tilde{R}$. The evolution of working capital $R(V)$ depends on the evolution of the entrepreneur stake in the project V, which is determined by the optimal contract. If an entrepreneur's value V reaches the value $\tilde{V} = pf(\tilde{R})/(1-\beta)$, any agency and enforcement problem becomes irrelevant, as the entrepreneur has no debt and has accumulated enough deposit to self-finance her firm at the efficient level $\tilde{R}$ in all subsequent periods. An entrepreneur can reach this state by making enough transfers to the intermediary, given that she does not exit exogenously. However, under some conditions discussed below, the value of a constrained entrepreneur could fall below a level at which it is optimal for the intermediary to liquidate the firm with some probability.

5.1 The optimal contract in the benchmark model economy

Consider first the optimal contract when the outside value is such that $\mathcal{O} = 0$. Under this assumption, an entrepreneur who repudiates its debt diverts and consumes the cash flow $f(R)$ and loses access to the capital market in the subsequent periods. The optimal contract in this case is similar to the one studied by Clementi and Hopenhayn [2006]. To be sure, Clementi and Hopenhayn [2006] do not consider limited enforcement in their model with private information. However, an enforcement constraint is implicitly satisfied under limited liability when $\mathcal{O} = 0$. Under risk neutrality, it is optimal to set repayment such that $\tau = f(R(V))$ whenever $V^H(V) \leq \widetilde{V}$, as it allows for the fastest accumulation of equity toward the unconstrained level.[10] The incentive compatibility constraint simplifies to

$$\beta V^H \geq f(R) + \beta V^L \ , \tag{20}$$

confirming that the enforcement constraint never binds when $\mathcal{O} = 0$ under limited liability as long as the incentive compatibility constraint is satisfied.[11]

Combining the incentive compatibility and promise-keeping constraint implies that the promised continuation values evolve according to $\beta V^L(V) = V - pf(R(V))$ when $V \leq \tilde{V}$ and $\beta V^H(V) = V + (1-p)f(R(V))$ when $V^H(V) \leq \tilde{V}$—that is, the intermediary optimally decreases an entrepreneur's value after low cash flow and increases her value otherwise, and $V^L < V < V^H$ for all $V^H(V) \leq \tilde{V}$. Conditional on not exiting exogenously, the firm grows on average when $V^H(V) \leq \tilde{V}$, as $\mathrm{E}(V'|V) = pV^H + (1-p)V^L = V/\beta > V$ for all $V^H(V) \leq \tilde{V}$. Feasibility and incentive compatibility imply that the period working capital $R(V)$ is determined by $f(R(V)) \leq \beta(V^H(V) - V^L(V))$. Clementi and Hopenhayn [2006] show that $R(V)$ is increasing in V for $V < \tilde{V}$ except in the neighborhood of the liquidation region, and that a constrained entrepreneur always receives less than the efficient level of working capital, $R(V) < \tilde{R}$ for all $V < \tilde{V}$. The firm either reaches the unconstrained level $\widetilde{V}$ after experiencing a sufficiently long but

[10] To see this, note that setting $\tau = f(R)$, the participation constraint implies that $V/\beta = pV^H + (1-p)V^L$, so that the entrepreneur's value grows at the maximum feasible rate.

[11] Recalling that debt repayment requires that $\beta V^H(V) \geq f(R)$ and limited liability requires that $V^L(V)$ and $V^H(V)$ be non-negative.

finite sequence of high-revenue shocks or faces liquidation with a positive probability whenever V falls below the threshold V_C. In the event of liquidation, it is optimal for the intermediary to set $Q = 0$.

Consider now the more general case of the benchmark contract for which $\mathcal{O} > 0$. Many results established in Clementi and Hopenhayn [2006] and reviewed above carry through when $\mathcal{O} > 0$. In this case, the enforcement constraint imposes a lower bound on the expected continuation values, which restricts the range of values $V^H(V)$ and $V^L(V)$ the intermediary can use to discipline the moral hazard.[12] To see this effect, note that preventing default requires that $pV^H(V) + (1-p)V^L(V) \geq pf(R) + \mathcal{O}$. A higher $\mathcal{O}$ implies a lower $R(V)$ for a given $V < \tilde{V}$, as $f(R(V)) \leq \beta(V^H(V) - V^L(V))$ must hold to maintain incentive compatibility. This implies the joint surplus $\hat{W}(V)$ is also lower for all $V < \tilde{V}$. In other words, a higher outside value $\mathcal{O}$ requires the entrepreneur to build a greater stake in her firm to obtain the same level of financing until she becomes unconstrained. Lastly, there will be no liquidation in equilibrium if the liquidation threshold is such that $V_C < \mathcal{O}$, as it is not feasible to promise continuation values that are less than $\mathcal{O}$.[13]

5.2 The effects of financial reforms on the optimal contract

In the benchmark economy, the costs of investing in enforcement capacity and monitoring firms are so high they are never incurred in equilibrium by the intermediary. The rest of this section considers the effects of reforms designed to improve contract enforcement and reduce private information by reducing the cost of investing in enforcement capacity and the cost of monitoring firms, respectively.

[12]Imposing a lower bound on the continuation value in this problem has been studied by Quadrini [2004] in the context of renegotiation-proof contracts. Although Quadrini [2004] does not consider limited enforcement, imposing a lower bound on expected continuation values is analogous to imposing an enforcement constraint if the lower bound is interpreted as the entrepreneur outside value. consequently, proofs of the results established by Quadrini [2004] are not repeated.

[13]Because the liquidation cutoff V_C satisfies $\hat{W}'(V_C) = (\hat{W}(V_C) - S)/V_C$, it follows that a lower $\hat{W}'(V)$ for all $V < \tilde{V}$ implies that V_C is larger. However, whether the increase in the liquidation threshold lead to an increase in equilibrium liquidation depends on the size of $\mathcal{O}$.

5.2.1 Improving contract enforcement

Assume the intermediary can invest $\phi(e)$ at the beginning of a period to decrease the entrepreneur's outside value to $\mathcal{O} - e$, where e denotes the level of enforcement.[14] That is, although the intermediary cannot recover the diverted cash flow, he can make it more difficult for an entrepreneur to finance a new firm after repudiating the contract. Assume further the enforcement cost function $\phi(\cdot)$ is strictly increasing, convex, and satisfies $\phi(0) = 0$. The implicit assumption in the benchmark contract is that $\phi(e)$ is so high for all $e \in (0, \mathcal{O}]$ that it is always optimal for the intermediary to choose $e(V) = 0$ for all V. When $\phi(e)$ is low enough, the choice of e enters the contracting problem such that conditional on the firm avoiding liquidation at the start of a period, the optimal contract maximizes the value of the joint surplus such that:

$$\widehat{W}(V) = \max_{\tau,R,V^L,V^H,e} pf(R) + (1-\delta)k(R) - (1+r)(R + \phi(e)) + \beta(pW(V^H) + (1-p)W(V^L))$$

$$\text{s.t. } (8), (9), (7), \text{ and } (11)$$

and the enforcement constraint

$$\beta V^L \geq \mathcal{O} - e \ . \tag{21}$$

The enforcement constraint (21) is the key difference relative to the benchmark contract. When the enforcement constraint binds, rearranging the first-order conditions and the Envelope condition yields:

$$f'(R)\left[\phi'(e)(1+r) + (1-p)\left(W'(V^L) - W'(V)\right)\right] = pf'(R) + (1-\delta)k'(R) - (1+r) \ , \tag{22}$$

where $V^L = \mathcal{O} - e$. Equation (22) highlights the dual effect of investing in enforcement capacity under private information. On the one hand, greater investment in enforcement capacity reduces small entrepreneurs' incentive to default. On the other hand,

[14] This assumption follows Popov [2014] who studies optimal enforcement decision in a dynamic lending contract with limited enforcement without private information.

it increases the range of continuation values the intermediary can use to discipline the moral hazard. In equilibrium, optimal enforcement $e(V)$ and the period working capital $R(V)$ are set such that, conditional on V, the marginal value of maintaining the credit relationship—the right-hand side of equation (22)—is equal to the marginal benefit of insuring the period cash flow against default plus the marginal benefit of relaxing the constraint on the low promised continuation value—the left hand side of equation (22).

Moreover, the first order conditions and the Envelope condition also yields

$$(1+r)\phi'(e) = W'(V) - \left[(1-p)W'(V^L) + pW'(V^H)\right] \ , \tag{23}$$

which highlights the two important margins of the optimal contract with private information and endogenous enforcement. Equation (23) shows that by investing in enforcement capacity, the intermediary is willing to accept lower debt repayment from small firms in the current period because it can promise them continuation values that maintain incentive compatibly and debt repayment in the next period. This new mechanism is at the heart of the complementarities between reforms investigated in Section 6.

Proposition 3 shows that the intermediary invests more in enforcement capacity when the entrepreneur's equity is lower and her incentive to repudiate the contract is higher. When the entrepreneur value becomes larger than the outside value option—i.e., when $V > \mathcal{O}$—the intermediary stops investing in enforcement capacity and only disciplines the moral hazard by promising high and low continuation values that are contingent on the cash flow. Given that V grows on average, Proposition 3 implies that investment in enforcement decreases with firm age.

Proposition 3 *When V is sufficiently low and the enforcement constraint binds:*

1. $e(V) > 0$

2. *increasing e permits lending a higher R for a given V*

3. *$e(V)$ is generally decreasing in V, but not necessarily monotonic*

Consider now the effect of a reform that lowers the enforcement capacity cost function such that $\phi(e) > \tilde{\phi}(e)$ for all $e > 0$. From equation (22), a lower enforcement cost increases the optimal $e(V)$ and decreases $V_L(V)$, as $\phi'(e) > \tilde{\phi}'(e)$ for all $e > 0$, which relaxes the constraint on $R(V)$. In the limit in which investing in enforcement is costless and $\phi(e) = 0$ for all $e > 0$, the contract once again reduces to the one studied by Clementi and Hopenhayn [2006], as e can be set to $\mathcal{O}$ at no cost.

5.2.2 Reducing private information

Starting again from the benchmark contract, assume now that the intermediary can monitor firm cash flow by incurring a fixed cost m at the beginning of each period. Paying m lets the intermediary verify the authenticity of a cash flow report but does not let him recover any diverted cash flow in the event that the entrepreneur misreports. An implicit assumption in the benchmark contract is that m is so high that the intermediary never chooses to monitor the firm in equilibrium. Consider now the case in which m is low enough to be relevant. Conditional on no liquidation taking place and on paying m at the beginning of a period, the value of the joint surplus is given by

$$\widehat{W}_M(V) = \max_{\tau,R,V^L,V^H} pf(R) + (1-\delta)k(R) - (1+r)(R+m) + \beta(pW(V^H) + (1-p)W(V^L))$$
$$\text{s.t. (8), (10), (7), and (11)}, \tag{24}$$

where the subscript M indicates *Monitoring*. It is optimal for the intermediary to set a misreporting entrepreneur's promised continuation value to zero and end the contract, as no other continuation values can increase the joint surplus. Consequently, V^L in the maximization problem above refers to the promised continuation awarded to an entrepreneur that truthfully report low cash flow. This property implies that the incentive compatibility constraint is satisfied as long as the enforcement constraint holds when the cash flow can be verified. The value of the joint surplus conditional on not paying m is given by

$$\widehat{W}_{NM}(V) = \max_{\tau,R,V^L,V^H} pf(R) + (1-\delta)k(R) - (1+r)R + \beta(pW(V^H) + (1-p)W(V^L))$$
$$\text{s.t. (8), (9), (10), (7), and (11)}, \tag{25}$$

where the subscript NM indicates *No Monitoring*. In this case, the incentive compatibility constraint is as relevant as in the benchmark contract.

Before discussing the optimal monitoring decision, consider first the limiting case in which $m = 0$. In this case, it is optimal for the intermediary to monitor an entrepreneur that reports low cash flow. Combining the participation and enforcement constraints yields

$$\beta V^H \geq f(R) + \mathcal{O} \text{ , and } \beta V^L \geq \mathcal{O} \text{ ,} \tag{26}$$

and feasibility requires that $V \geq \mathcal{O}$. It follows that, when monitoring firm is costless, the only relevant friction is limited enforcement, and the contract is similar to the one studied by Albuquerque and Hopenhayn [2004]. It is well-known that in this case, it is optimal for the intermediary to maintain an entrepreneur that truthfully reports low cash flow at the same size such that $V^L(V) = V$. Furthermore, the participation constraint implies that $V^H(V) = cV$ for $V^H(V) < \widetilde{V}$ where $c = \frac{1-\beta p}{\beta(1-p)} > 1$ when $\beta < 1$ so that an entrepreneur's value increases after reporting high cash flow. Consequently, a firm may never face liquidation if the entrepreneur's initial stake is greater than the liquidation threshold, such that $V_0 > V_C$. As in the benchmark contract, firms grow on average, as $\mathbb{E}(V'|V) = pcV + (1-p)V > V$ for all $V^H(V) \leq \tilde{V}$. The state contingent period loan $R(V)$ is determined by

$$R(V) = \begin{cases} f^{-1}(cV - \mathcal{O}) & \text{if } V < V_u < \tilde{V} \\ \tilde{R} & \text{if } V_u \leq V \leq \tilde{V} \end{cases} \text{ ,} \tag{27}$$

where V_u is such that $f^{-1}(cV_u - \mathcal{O}) = \widetilde{R}$. The value V_u is the minimum equity at which an entrepreneur can obtain the unconstrained level of financing from the intermediary and is generally less than the unconstrained level $\widetilde{V}$. Although the firm operates at its efficient scale when $V \in [V_u, \widetilde{V})$, the entrepreneur still requires financing from the intermediary until V reaches $\widetilde{V}$.

When the cost of investing in enforcement capacity $\phi(\cdot)$ is low enough, the contract with costless monitoring $m = 0$ corresponds to a simple version of the one studied by

Popov [2014]. In this case, the joint surplus conditional on no liquidation is

$$\widehat{W}_M(V) = \max_{\tau,R,V^L,V^H,e} pf(R)+(1-\delta)k(R)-(1+r)(R-\phi(e))+\beta(pW_M(V^H)+(1-p)W_M(V^L)) \tag{28}$$

subject to constraints (8) and (11), the enforcement constraints

$$\beta V^H \geq f(R) + \mathcal{O} - e \text{ and } \beta V^L \geq \mathcal{O} - e \ , \tag{29}$$

and the feasibility constraint $V \geq \mathcal{O} - e$. Optimal enforcement in this case is determined in equilibrium by

$$f'(R)\phi'(e)(1+r) = pf'(R) + (1-\delta)k'(R) - (1+r) \ . \tag{30}$$

Equation (30) is the analog of equation (22) and shows that investing in enforcement capacity only acts as insurance against repudiation when monitoring firms is costless. As before, the tradeoff between the cost of enforcement and the debt repayment is given by

$$(1+r)\phi'(e) = W'(V) - \big((1-p)W'(V^L) + pW'(V^H)\big) \ , \tag{31}$$

noting that $e(V)$ is strictly decreasing in V in this case, as $V^L(V) = V$.[15] Proposition 4 shows that it is optimal for the intermediary to invest in enforcement capacity e as long as the firm operates below its efficient level, and e is strictly decreasing in V in this case. Moreover, Proposition 4 implies that e decreases with firm age, as V grows on average.

Proposition 4 *Given V such that $R(V) < \tilde{R}$*

1. *$e(V) > 0$, and 0 otherwise*

2. *increasing e permits lending a higher R for a given V*

3. *$e(V)$ is strictly decreasing in V*

[15] See the proof of Proposition 4 in the appendix for more details.

I now turn to the optimal monitoring decision. From the above discussion, when $m = 0$, $\widehat{W}_M(V) > \widehat{W}_{NM}(V)$ for $V < \tilde{V}$, as $\widehat{W}_M(V)$ and $\widehat{W}_{NM}(V)$ are strictly concave and constrained firms operate at their efficient scale when $V_u < \tilde{V}$. A higher m decreases the joint surplus $\widehat{W}_M(V)$ for all $V \in (0, \tilde{V}]$ because it decreases the period expected profit. In the limit, m is so high that $\widehat{W}_{NM}(V) > \widehat{W}_M(V)$ for all $V \in (0, \tilde{V}]$, the contract corresponds to the benchmark, and m is never incurred. Because $\widehat{W}_M$ is more concave than $\widehat{W}_{NM}$, there is a range of values for $m > 0$ for which $\widehat{W}_M$ and $\widehat{W}_{NM}$ intersect at two points V^{low} and V^{high} such that $V^{low} < V^{high}$ and $\widehat{W}_M(V) > \widehat{W}_{NM}(V)$ for all $V \in (V^{low}, V^{high})$. To make some progress in characterizing the optimal monitoring decision, I assume that the initial value of the contract is such that $V^{low} < V_0 < V^{high}$ and concentrate on the monitoring decision around V^{high}.[16] Under this assumption, the value of a new firm does not decrease following a truthful low-revenue report because the intermediary commits to monitoring—i.e., $\widehat{W}_M(V_0) > \widehat{W}_{NM}(V_0)$—and thus never reaches V^{low}. As with the liquidation decsion, it is optimal for the intermediary to randomize the monitoring decision such that:[17]

$$\widehat{W}(V) = \max_{\varsigma \in \{0,1\}, V_M, V_{NM}} \quad \varsigma \widehat{W}_M(V) + (1-\varsigma)\widehat{W}_{NM}(V) \qquad \text{s.t. } \varsigma V_M + (1-\sigma)V_{NM} \geq V \ , \tag{32}$$

where $\varsigma(V)$ is the probability that the intermediary commits at the beginning of the period to monitor the firm cash flow, and V_M and V_{NM} are the promised continuation values awarded to the entrepreneur given that the intermediary commits to monitor or not, respectively. Note that although $\varsigma(V)$ does not depend on the ex-post cash flow realization during the period, it does depend on the financial position V of the firm at the start of the period.

In equilibrium, a firm that starts at $V_0 < V_M$ is monitored each time its entrepreneur reports low cash flow. If the cost of enforcement capacity $\phi(\cdot)$ is not too high, the in-

[16] Although this assumption is not without loss of generality, it was found to hold in all numerical exercises.

[17] Note that although I do not discuss the randomization around V^{low}, the optimal value function needs to be solved numerically by considering the randomization of the monitoring decision around both V^{low} and V^{high}.

termediary chooses how much to invest according to equation (30). Conditional on not exiting exogenously, the firm grows on average and Proposition 4 shows that the intermediary invests less in enforcement capacity over time. Given a long enough sequence of high cash flow, the firm passes the threshold V_M and enters the randomization region. With probability $1 - \varsigma(V)$, the firm continues at V_{NM} in the next period from which point the intermediary stops monitoring the firm and disciplines the moral hazard by promising high and low continuation values. The intermediary does not invest in enforcement capacity past this point because only the incentive compatibility constraint continues to bind for $V > V_{NM}$. If the firm experiences a long enough sequence of low-revenue shock, it may re-enter the monitoring lottery region but can never decrease in size below V_M. With a long enough sequence of high-revenue shocks, the firm reaches the unconstrained level.[18]

6 Aggregate consequences of financial reforms

In this section, I estimate the parameters of the benchmark economy using data from Colombia in the 1980s and early 1990s. This choice of benchmark is motivated by World Bank research and data availability. The World Bank has identified a number of legal and institutional factors in Colombia in the 1980s and early 1990s that prevented efficient firm monitoring and financial contract enforcement. For example, infrastructures facilitating consistent and accurate corporate financial reporting were lacking at that time, which prevented efficient firm monitoring.[19] In addition, the heavy bias against secured creditors during insolvencies, the lack of effective collection mechanisms for creditors, and the protracted insolvency proceedings contributed to inefficient

[18] This version of the contract with stochastic monitoring is related to Monnet and Quintin [2005] who consider the role of costly state verification and stochastic monitoring in a dynamic lending contract with fixed firm size.

[19] Part of the reason was the absence of prerequisite qualification for registration as a public accountant and the multiple and often contradictory source of accounting standards [World Bank, 2003]. In addition, although the majority of businesses were legally required to appoint a *revisor fiscal* for conducting annual audits, the *revisor fiscal* was also legally responsible for the quality of its client company's internal control, thereby precluding an actual external audit [World Bank, 2003].

enforcement of contracts.[20] Against this institutional backdrop, the availability of detailed establishment-level data constructed by Eslava, Haltiwanger, Kugler, and Kugler [2004, 2013] makes Colombia in the 1980s and early 1990s an ideal laboratory to study the effects of different types of financial reforms. I then simulate the model to investigate the effects of a series of counterfactual reform experiments deigned to reduce private information and limited enforcement frictions.

6.1 Parameterization of the benchmark model economy

The model is parameterized as follows. The worker disutility from labor is

$$\varphi(h) = A \cdot h^{\frac{1+\eta}{\eta}} , \tag{33}$$

where η is the elasticity of labor. The firm production function is

$$f(k, n) = z(k^{\xi} n^{1-\xi})^{\theta} , \tag{34}$$

where z is a scaling parameter.

The interest rate r is set to 0.12, which is roughly the average real lending rate during the 1980s and early 1990s documented by Bond, Tybout, and Utar [2015] and implies a discount rate for the workers around 0.89. The mass of workers λ is normalized to 1 and A is chosen so that hours worked are 0.36 to be consistent with the average for Colombia around that time. Following Atkenson, Khan, and Ohanian [1996] and Cooley et al. [2004], I set $\theta = 0.85$. This value implies $\xi = 0.26$ for unconstrained firms given that the labor share in Colombia in the 1980s and early 1990s is about 0.63. The

[20] For example, estimating in advance the degree of coverage granted by the collateral was almost impossible in Colombia because mortgages and pledges could only be recovered after the debtor's labor and tax claims were fully paid with proceeds [World Bank, 2006]. In addition, majorities made up of unsecured creditors and shareholders of the insolvent firm could often alter their priorities and modify the conditions of their claims without the secured creditors' consent [World Bank, 2006]. Moreover, prior to reforms in 1995, mandatory liquidations were handled by civil courts lacking capacity, and business knowledge could last many years [Xavier Gin, 2010]. A contributor to the significant delays in claim collection was that debtors in Colombian executory proceedings were not limited to raising only specific defenses [World Bank, 2006]. Lastly, the legal framework and financial information infrastructure were not sufficiently developed to support efficient credit reporting and sharing of creditor information [Miller and Gadamillas, 2006].

workers' elasticity of labor is set to 1. Lastly, the parameter κ is chosen to ensure that the contract is feasible, and z is set to normalize the unconstrained level of working capital $R(\tilde{V})$ to 1.

The remaining parameters are the capital depreciation rate δ, the parameter p that governs the cash flow shock process, the fixed initial investment I_0, the salvage value S, and the exogenous exit rate γ. I first estimate δ and p by exploiting the cross-sectional and time series variations in firm-level data on the Colombian manufacturing sector between 1982 and 1992.[21] The parameters I_0 and S have an important effect on the contract and on firm dynamics but do not have a direct counterpart in the data. To make some progress, I follow the recent dynamic corporate finance literature—e.g., Hennessy and Whited [2005] and DeAngelo, DeAngelo, and Whited [2011]—and estimate structurally I_0, S and γ using a SMM procedure that chooses model parameter values that set moments of artificial data simulated from the model as close as possible to corresponding real data moments, conditional on the other calibrated and estimated parameters. Identification of the parameters by SMM comes from the property of the contract with respect to these parameters rather than cross-sectional and time-series variation in the data.[22]

Panels A through C of Table 1 summarize the parameterization of the benchmark economy. Most of the entries are self-explanatory, but a few require an explanation. The value of p is the average parameter value of a two-point approximation of the distribution of firm-level total factor productivity estimated by Eslava et al. [2004], where the average and standard deviation are taken across the three-digit manufacturing industries. As explained in the Appendix, a useful property of the optimal contract that facilitates the SMM estimation is that it becomes invariant to the wage rate w if z is set so that $R(\tilde{V}) = 1$, I_0 is expressed as a fraction of the maximum joint surplus $W(\tilde{V})$, and S is expressed as a fraction of I_0. Using this normalization, the parameter estimate

[21] The data used to estimate the contract parameters are a subset of the database constructed by Eslava et al. [2004] from the Colombian Annual Manufacturing Survey for 1982 to 1998. The data set used in this analysis contains information on the capital stock (buildings/structures and machinery/equipment) and total factor productivity of establishments in 29 three-digit manufacturing sectors that were operating between 1982 and 1992. See the Appendix for details.

[22] I explain the details of the SMM estimation procedure and the choice of moments in the Appendix.

of I_0 implies that the fixed investment cost is 12.7 percent of the maximum joint surplus such that $I_0 = 0.127 \times W(\tilde{V})$. Similarly, the value for S implies that 3.6 percent of the initial fixed investment is sunk, such that $S = 0.964 \times I_0$. The t-statistics in Panel C suggest that the simulated moments match the actual moments well and that the model can also account reasonably well for the mean of small firms' investment-to-capital ratio that was not targeted by the SMM procedure.

6.2 Steady-state allocations and firm dynamics

Table 2 summarizes the results from two counterfactual reform experiments that completely eliminate one of the two sources of financial frictions in the benchmark economy. These exercises assume that the reformed economies reached their new steady state and their transition dynamics are investigated in the next sub-section. The top half of Table 2 reports values for aggregate output, consumption, hours, and the wage rate in the benchmark and the two reformed economies expressed as percentages of first-best values. The bottom half of Table 2 reports the firm liquidation rate—i.e., the endogenous exit rate—the size of the new and average firm measured by its capital stock divided by the unconstrained level, and the fraction of firms operating at their efficient scale.[23]

Column 1 of Table 2 shows that financial frictions in the benchmark economy lead to a 16.7 percent loss in aggregate output relative to the potential first-best level. Columns 2 and 3 show that while limited enforcement by itself leads to a 7.8 percent loss in aggregate output, its relative effect in the presence of private information frictions is small—that is, private information by itself leads to a 14.9 percent output loss, or nearly 90 percent of the aggregate output loss in the benchmark economy.

This result stems from the property that the enforcement constraint under private information only binds for the smallest firms in the economy while the incentive

[23]Note that the entry/exit rate, which is the sum of the liquidation rate and the exogenous exit rate γ, is slightly higher than the value reported in Panel C of Table 1. This difference arises because the exogenous exit rate γ is estimated rather than calibrated and the estimated liquidation rate in the (finite sample) SMM procedure is different from the (asymptotic) solution of the model. For instance, the SMM procedure uses moments computed from a finite number of artificial data sets that each contain the lifecycles of the same number of firms as in the actual data set. In contrast, when solving for the general equilibrium, I estimate the ergodic distribution of promise utilities.

constraint binds for all firms requiring external financing. Consequently, private information frictions create a larger distortion in terms of aggregate output by creating a greater mass of inefficient firms. In addition, Section 5 showed that eliminating enforcement frictions leads to two effects. First, firms can obtain a greater level of financing for the same level of equity. Second, the intermediary can maintain poor-performing firms at a smaller size while still maintaining incentive compatibility. In general equilibrium, a reform that eliminates enforcement frictions without reducing private information frictions leads to a wider distribution of firms and a higher liquidation rate, which, together with a higher wage rate, only leads to a modest increase in steady aggregate output. In contrast, a reform that eliminates private information frictions leads to a substantial increase in the fraction of firms that can operate at their efficient scale.

Figures 2 and 3 illustrate this mechanism further by summarizing the effects of each reform on firm dynamics. Figure 2 plots the mean and standard deviation of firm growth conditional on firm age and Figure 3 plots the average firm size conditional on firm age. When private information frictions are eliminated, young firms grow faster on average, their growth is less volatile and virtually all 20-year-old firms operate at their efficient scale. In contrast, the average 20-year-old firm operates at about 80 percent of its efficient scale in the steady state where enforcement frictions were eliminated.

6.3 Transition dynamics of the reformed economies

This section investigates the transition dynamics of the benchmark economy in the years following a reform that eliminates limited enforcement and private information frictions. In these exercises, a reform is unexpected and permanent. The reform is implemented at the end period t_0 and after the promised continuation values are awarded to entrepreneurs. At the beginning of the next period, t_0+1, the optimal contract takes as given the sequence of equilibrium prices $\{w_\iota\}_{\iota=t_0+1}^{\infty}$ to solve

$$\begin{aligned} W(V, \{w_\iota\}_{\iota=t}^{\infty}) = \max_{\alpha\in[0,1],Q,V_C} \quad & \alpha S + (1-\alpha)\widehat{W}(V_C, \{w_\iota\}_{\iota=t}^{\infty}) \\ & \text{s.t. } \alpha Q + (1-\alpha)V_C \geq V \ , \end{aligned} \tag{35}$$

where

$$\widehat{W}(V,\{w_\iota\}_{\iota=t}^{\infty}) = \max_{R,\tau,V^H,V^L} pf(R)+(1-\delta)k(R)-(1+r)R \\ +\beta(pW(V^H,\{w_\iota\}_{\iota=t+1}^{\infty})+(1-p)W(V^L,\{w_\iota\}_{\iota=t+1}^{\infty}))\ , \tag{36}$$

subject to the promise-keeping constraint (8), the feasibility constraint (7), and additional constraints that depend on the type of reform implemented.[24] When private information frictions are eliminated, debt repayment requires that $f(R)-\tau+\beta V^H \geq f(R)+\mathcal{O}(\{w_\iota\}_{\iota=t_0+1}^{\infty})$ and $V^L \geq \mathcal{O}(\{w_\iota\}_{\iota=t_0+1}^{\infty})$, noting that limited liability is satisfied if these constraints are satisfied. When the contract is fully enforceable, incentive compatibility and limited liability require that $f(R)-\tau+\beta V^H \geq f(R)+\beta V^L$ and $V^L \geq 0$. Given the sequence of equilibrium prices $\{w_\iota\}_{\iota=t_0+1}^{\infty}$, the optimal contract implies a sequence of firm distribution $\{\mu_\iota\}_{\iota=t_0+1}^{\infty}$ that is consistent with market clearing and is taken into account by workers when solving their consumption problem.

Figure 4 plots the transition of aggregate output and the aggregate investment-to-capital ratio expressed in percentage deviation from the benchmark economy level in the 30 years following each reform. Figure 4 shows that eliminating private information frictions leads to roughly 11 consecutive years of growth at an average growth rate of about 1 percent per year. This growth is associated with a higher capital accumulation in the 10 years after private information is eliminated. In contrast, eliminating limited enforcement frictions leads to an immediate 1.5 percent rise in aggregate output and less than 0.5 percent growth in the subsequent 30 years. Given the assumption on the agents' preferences, the increase in output is proportional to the increase in wage and

[24]Proposition 2 guarantees the existence and uniqueness of a post-reform steady state. Proving the existence of an equilibrium in the transition between steady states is more difficult and would require a generalization of the proof given in the Appendix for the steady state equilibrium. While I do not provide a formal proof in this paper, the argument would consist of showing (i) that the value function of the contract is well-defined given a sequence of prices, (ii) that given a Cauchy sequence of prices from a set of Cauchy sequences, there exists a mapping based on market clearing that returns a Cauchy sequence of prices from the same set, and (iii) that the new Cauchy sequence of prices returned by this mapping is closer at every point in the sequence to the previous Cauchy sequence of prices. The numerical algorithm discussed in the Appendix conjectures this is argument is valid, and the iterative convergence of the algorithm that solves for the trajectory of prices is suggestive that such an equilibrium may exist. The Appendix also contains a definition of the equilibrium along the transition path.

hours following the reform.[25]

6.4 Welfare gains

To investigate the role of each reform's transitory effects on welfare, Table 3 contrasts the welfare gains, taking into account transitional dynamics to the welfare gains calculated across steady states. Recalling that the financial intermediary is owned by a coalition of entrepreneurs, welfare in a steady state is the sum of the workers' wealth plus the aggregate joint surplus generated by the firms in the economy minus the aggregate sunk cost associated with setting up new firms—that is,

$$\text{Stready state welfare} := \frac{wH(w) - \varphi(H(w))}{r} + \int W(V)d\mu - \frac{\hat{\gamma}(I_0 - S) + \gamma I_0}{r} \quad (37)$$

where $\hat{\gamma}$ denotes the fraction of firms that are liquidated in each period. Similarly, aggregate welfare, taking into account the transition dynamics of the economy, is defined as

$$\begin{aligned} \text{Dynamic welfare} := & \sum_{\iota=t_0+1}^{\infty} \left(\frac{1}{1+r}\right)^{\iota} [w_\iota H(w_\iota) - \varphi(H(w_\iota))] \\ & + \int W(V, \{w_\iota^i\}_{\iota=t_0+1}^{\infty}) d\mu_{t_0} - \sum_{\iota=t_0+1}^{\infty} \left(\frac{1}{1+r}\right)^{\iota} (\hat{\gamma}_\iota (I_0 - S) + \gamma I_0) \end{aligned} \quad (38)$$

where μ_{t_0} is the distribution of promised value at the end of period t_0 when the reform is implemented. Computing welfare gains consists of subtracting the benchmark steady-state welfare from these quantities computed for the two different reforms.

Table 3 shows that, regardless of whether one accounts for transitory effects, eliminating private information frictions delivers aggregate welfare gains of about 7 percent, while eliminating enforcement frictions deliver aggregate welfare gains of less than 0.5 percent. This large difference in aggregate welfare gains is to be expected given the vastly different effects of each reforms on resource allocation discussed above. How-

[25]The Appendix provides more details about the general equilibrium effects on reallocation by computing the transition of aggregate output holding the wage rate fixed at its pre-reform level. The Appendix also plots the transition of the equilibrium wage rate, aggregate hours and capital stocks.

ever, focusing on the different groups of agents in the economy, Table 3 shows that entrepreneurs tend to lose from the private information reform in the long run but gains when accounting for transitory effects.

This curious result is due to entrepreneurial rent. Recall that there is a fixed number of blueprints in the economy that can only be turned into firms by entrepreneurs. Consequently, firms that operate below their efficiency level in the benchmark economy are very profitable. When private information frictions are eliminated, firms in the new steady state are much larger and entrepreneurial rent is much lower because firms must pay a higher wages to workers, resulting in negative welfare gain for entrepreneurs.

That said, the elimination of private information frictions yields positive gain for entrepreneurs if one takes into account for the transition dynamics of the economy. As dicussed above, the transition of the economy to the new steady state after private information frictions are eliminated is slow, which means that entrepreneurs continue to enjoy relatively high rent in the first ten years or so following the reform. Conversely, because the effects of eliminating enforcement frictions are more modest, the entrepreneurs' welfare gains are broadly similar, with and without accounting for transition dynamics.

6.5 Complementarities between reforms

This section concludes the quantitative analysis by investigating complementarities between reforms that reduce the cost of investing in enforcement capacity $\phi(\cdot)$ and of firm monitoring m. I measure complementarities by calculating the aggregate welfare gains in an economy that experienced a reform that jointly lowered the costs of enforcement and of monitoring and subtracting from this value the sum of the individual welfare gains associated with reducing only one of the two costs. I perform the same calculation with aggregate consumption and consider the effects of intermediate reforms that decrease the enforcement and monitoring costs to a non zero value and more extreme reforms that decrease these costs to zero.[26]

[26] This exercise requires solving for the steady state of six different economies in addition to the benchmark economy. Solving for the steady state in economies experiencing intermediate reforms

Table 4 shows that the welfare gains are between 1.3 and 1.8 percentage points higher when a reform jointly addresses private information and enforcement frictions. The results are broadly similar for aggregate consumption, with gains that are between 1.5 and 2.6 percent higher. These complementarities arise because of the properties of the contract discussed in Section 5. By investing in enforcement capacity, the intermediary is willing to accept lower debt repayment from small firms in the current period while promising them continuation values that maintain incentive-compatibly and debt repayment in the next period. Because young firms can obtain greater working capital on average when enforcement capacity is high, young entrepreneurs build more rapidly a greater stake in their firm and the moral hazard become less significant sooner. These complementarities are also reflected in the aggregate expenditures on monitoring and enforcement. For example, the sum of expenditures on monitoring and enforcement when both costs are jointly lowered to a non zero level is about 11 percent lower than the sum of expenditures on monitoring and enforcement when only one of these costs is reduced.

7 Conclusion

This paper investigates the effects of financial frictions on firm dynamics, aggregate resource allocation, and economic development by proposing and estimating a general equilibrium model in which different forms of financial frictions affect the supply of credit to firms. The optimal dynamic contract at the core of the model bridges the gap between many existing cases that arise under either limited enforcement or private information, and this paper analyzes new cases that arise under both forms of financial frictions. Estimating the model using data on Colombian manufacturing firms in the 1980s and 1990s, I find that financial frictions lead to a significant aggregate

requires solving numerically for the contract with endogenous enforcement and stochastic monitoring analyzed in Section 5. Note that there are still limited enforcement frictions when the cost of investing in enforcement capacity $\phi(e)$ is reduced to zero for all $e > 0$. In this case, a repudiating entrepreneur can still abscond with the cash flow but is permanently excluded from capital market—i.e., $\mathcal{O} = 0$. Moreover, the enforcement constraint in this case never binds with $m \to \infty$ and only binds for very small firms when $m \in \{0, 0.05\}$. As a result, the economy with zero monitoring and enforcement cost is very close to first-best.

output loss, and that most of this distortion can be attributed to private information frictions. Reforms that reduce private information frictions lead to economic growth and subtantial welfare gains, while reforms that improve contract enforcement without addressing private information frictions do not. I show that there are complementarities between the two types of reforms, as moral hazard problems tend to be less significant for financial contracting when contracts are easier to enforce.

The analysis suggests two potentially fruitful avenues for further research. First, the structure of the dynamic contract offers a viable starting point to identify the importance of different forms of financial frictions in data. For example, cross-industry and cross-country heterogeneity in firm and industry dynamics could be exploited to structurally estimate the relative importance of each form of financial frictions. Second, the model could be extended to investigate the amplification and propagation of aggregate shocks given different forms of financial frictions. Following the seminal work of Bernanke and Gertler [1989] and Kiyotaki and Moore [1997], a large literature developed to study the amplification and propagation of aggregate shocks under financial frictions—see Quadrini [2011] and Brunnermeier, Eisenbach, and Sannikov [2012] for surveys. A pervasive assumption in this literature is to restrict the horizon of financial arrangements to one period. This assumption places collateral at the center stage of credit allocation, which may amplify and propagate the effect of aggregate shocks under some conditions. When agents can contract over a longer horizon, lending may occur in equilibrium even if entrepreneurs do not have sufficient collateral. In this case, the nature of the financial arrangements could be an important determinant of aggregate fluctuations.

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8 Figures and tables

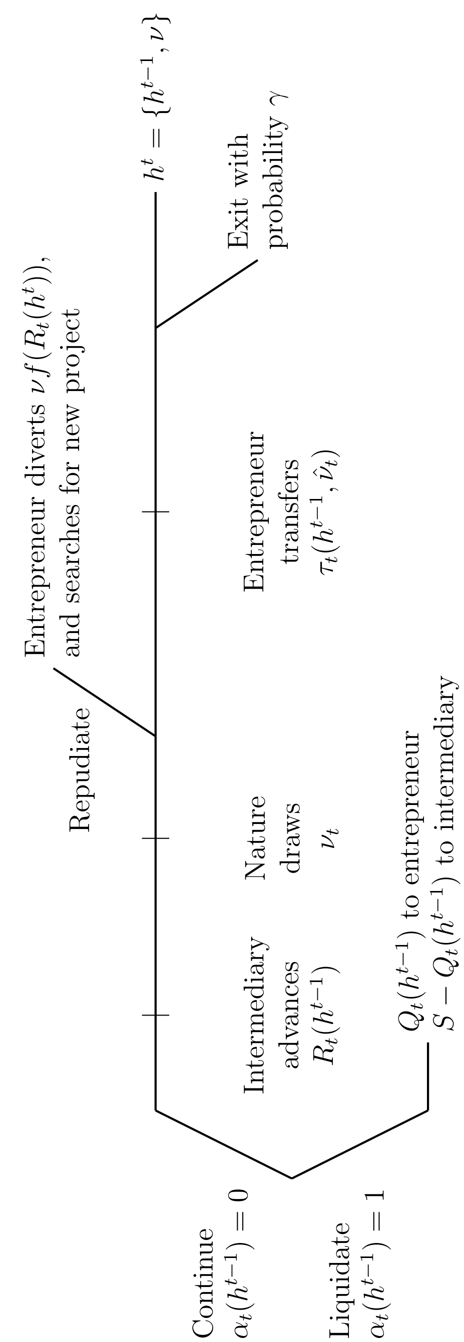
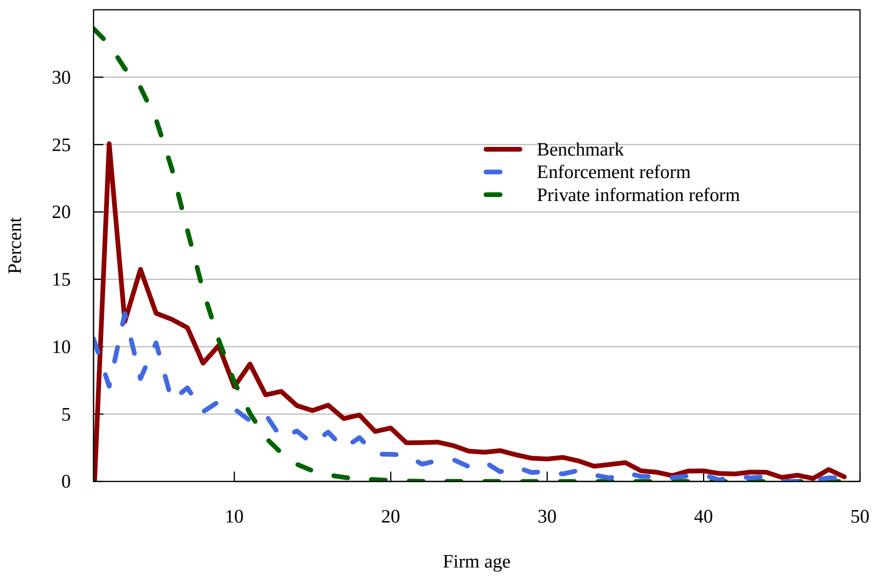


Figure 1: Timing within one period.



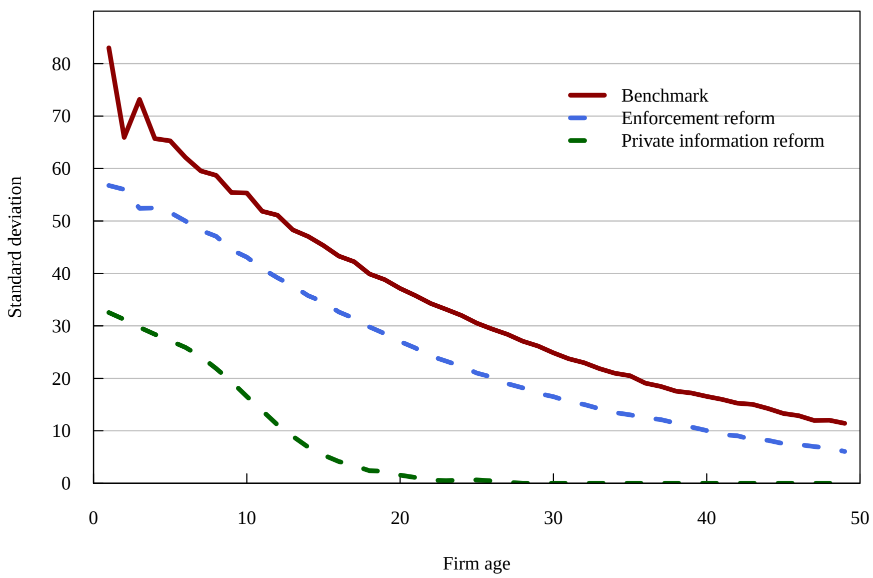


Figure 2: Firm investment growth rate (top) and standard deviation of firm investment growth rate (bottom) conditional on firm age.

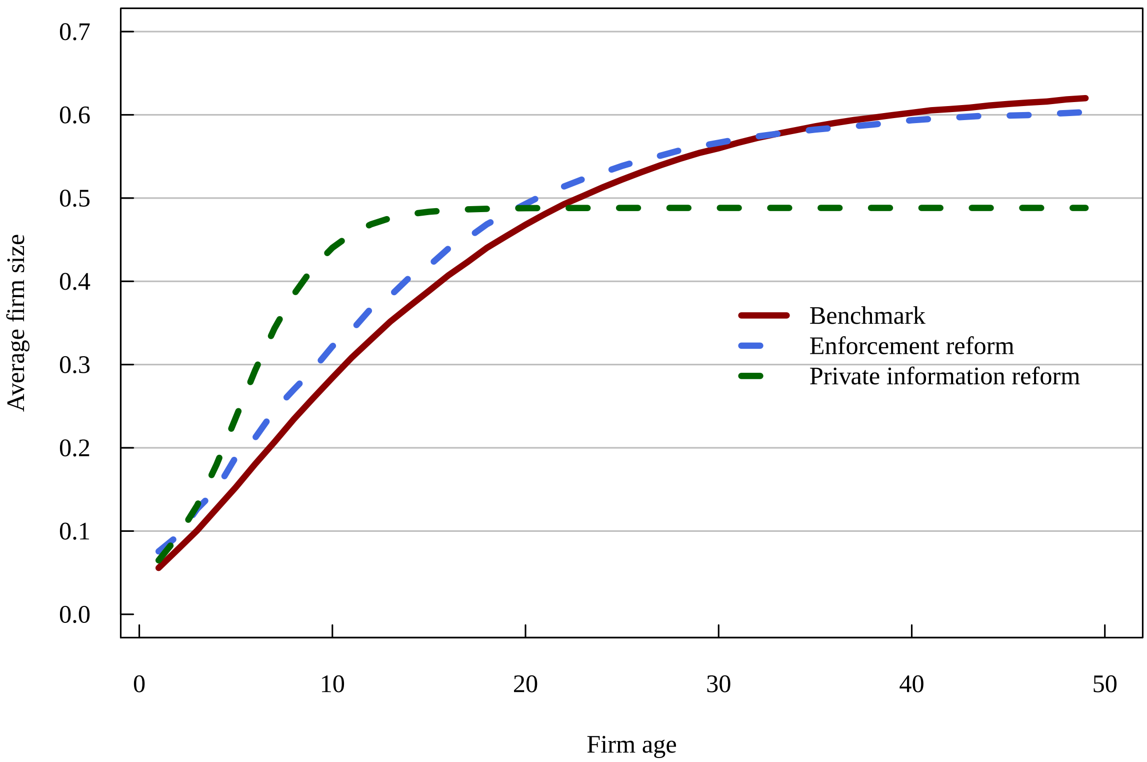


Figure 3: Average firm size in terms of capital conditional on firm age.

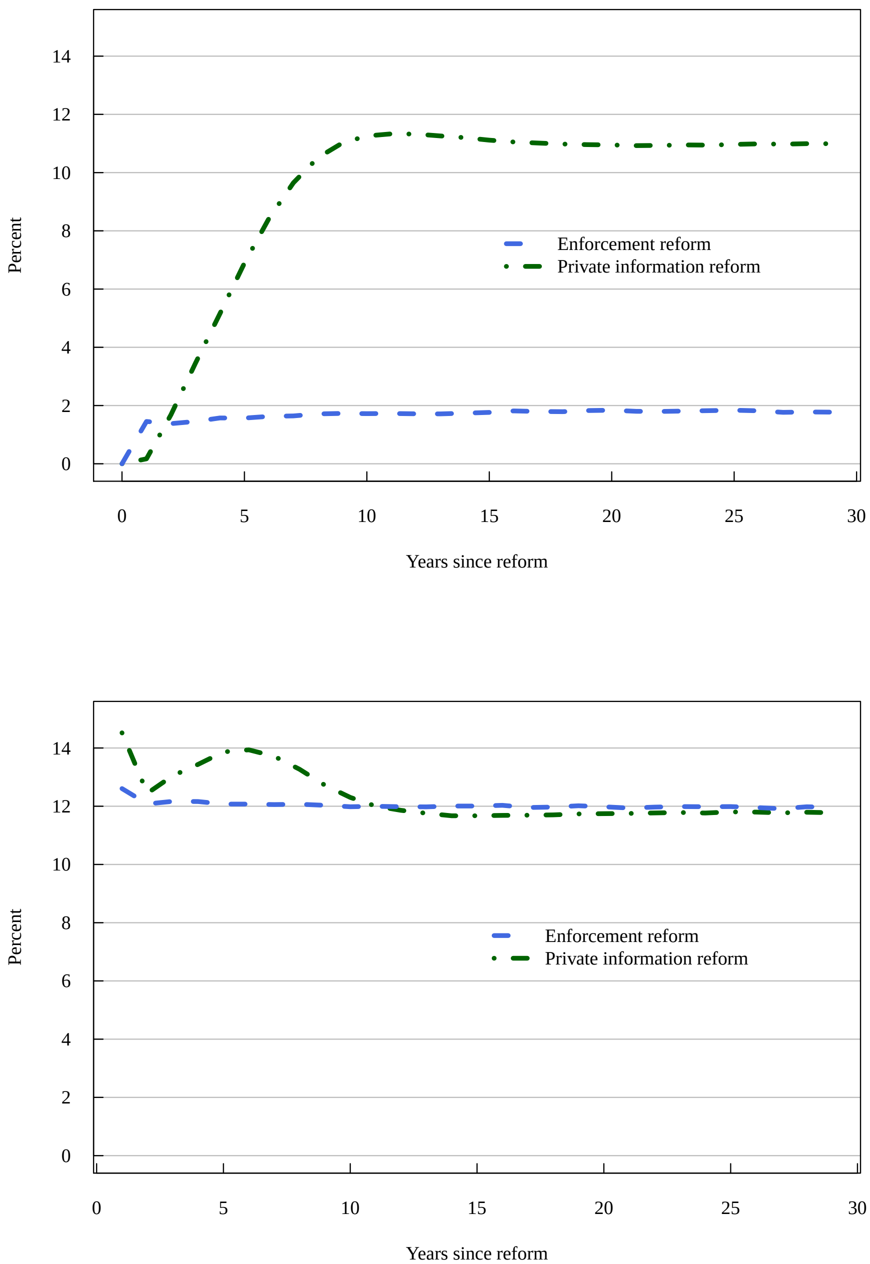


Figure 4: Aggregate output (top) and investment-to-capital ratio (bottom).

Table 1: Model parameters.

A. Calibration

	Parameters	Value
Interest rate	r	0.12
Worker utility function	A	0.617
	η	1
Firm production function	θ	0.85
	ξ	0.26
	z	1.533
Contract repudiation cost	κ	0.45

B. Panel estimation

	Parameters	Estimates	Standard errors
Depreciation rate	δ	0.105	0.025
Revenue shock	p	0.518	0.153

C. SMM estimation

	Parameters	Estimates	Standard errors
Fixed set up cost	I_0	0.127	0.013
Salvage value	S	0.964	0.019
Exogenous exit	γ	0.058	0.000
	Actual moments	Simulated moments	t-Statistics
Exit rate	0.063	0.062	0.059
Exit rate, small firms	0.12	0.12	-0.001
Exit rate, large firms	0.055	0.056	-0.195
Variance of investment, small firms	0.926	0.863	0.171
Average investment (untargeted)	0.32	0.343	-2.569

Table 2: Steady state allocation and firm dynamics.

	(1) Benchmark economy	(2) Frictions eliminated: Limited enforcement	(3) Frictions eliminated: Private information
	% relative to first best		
Wage rate	89.8	90.7	95.4
Aggregate output	83.3	85.1	92.2
Aggregate consumption	83.9	85.5	92.6
Hour worked	89.8	90.8	95.5
Liquidation rate (%)	1.1	2	0
Relative size of new firms (%)	6.6	9.6	9
Relative size of the average firm (%)	47.3	50.5	72.3
Fraction of unconstrained firms (%)	27.1	28.9	57

Table 3: Welfare gains of financial reforms (%).

	Frictions eliminated: Limited enforcement	Frictions eliminated: Private information
	steady state comparaison	
Worker	2.06	12.92
Entrepreneurs	-2.97	-3.61
Aggregate	0.29	7.12
	accounting for transition dynamics	
Worker	1.33	7.98
Entrepreneurs	-1.08	5.09
Aggregate	0.49	6.97

Table 4: Financial reform complementarities.

A.	*Welfare gains (%)*		
	Enforcement capacity cost		
Monitoring cost	$\phi(e) \to \infty$	$\phi(e) = \frac{1}{2}e^2$	$\phi(e) = 0$
$m \to \infty$	–	-1.39	0.26
$m = 0.05$	5.49	5.94	–
$m = 0$	7.12	–	8.72

B.	*Consumption gains (%)*		
	Enforcement capacity cost		
Monitoring cost	$\phi(e) \to \infty$	$\phi(e) = \frac{1}{2}e^2$	$\phi(e) = 0$
$m \to \infty$	–	0.33	1.76
$m = 0.05$	7.63	9.46	–
$m = 0$	10.34	–	14.74

A Appendix

A.1 Existence and uniqueness of a general equilibrium

The proof of existence is fairly general and does not rely on any particular assumption on the worker preferences and firm production function, besides those needed to have a well-defined contract value functions. Proving existence consists of establishing parametric continuity of the ergodic distribution of promised values using LeVan and Stachurski [2007, Theorem 1]. The proof of uniqueness, however, relies on the assumptions on the workers' preferences. For simplicity, I only prove existence and uniqueness for the case in which monitoring is not feasible and enforcement is the strongest—that is, $m \to \infty$ and $\phi(e) = 0 \forall e > 0$.[27]

Given interest rate r and wage w, perfect competition in the financial market implies that new firms start with equity V_0. Consider the sequence $(X_t)_{t\geq 0}$ of equity levels from a single firm indefinitely replaced by a new one upon liquidation or exogenous exit, with $X_0 := V_0$. It is clear that $(X_t)_{t\geq 0}$ is a sequence of random variables, and its evolution depends on the properties of the optimal contracts and on the sequence of shocks—liquidation lottery, cash flow shock, and exogenous exit.

The proof consists of five parts. The first part shows that $\mathbf{X} = (X_t)_{t\geq 0}$ is a time-homogeneous Markov chain such that

$$X_{t+1} = T_\omega(X_t, \epsilon_t), (\epsilon_t)_{t\geq 0} \sim \phi_\omega \in \mathscr{P}(Z), X_0 = V_0 \in S \tag{39}$$

where $T_\omega : S \times Z \to S$ is a collection of measurable functions indexed by $\omega \in \Omega$ the parameter space, $(\epsilon_t)_{t=1}^{\infty}$ is a sequence of independent random shocks with joint distribution ϕ_ω, and S and Z are the state space and the probability space, respectively. The second part of the proof establishes that this Markov chain admits a unique stationary distribution, which can be attained in a finite number of periods starting from any

[27] The proof is similar for the other cases but requires an extra step when V_0 affects the enforcement constraint. For these cases, it remains to be shown that for given prices, the mapping T defined in section 4 has a unique fixed point when V_0 affects the enforcement constraint. From Cooley et al. [2004], it is sufficient to show that the, given the mapping T is continuous, T is monotone decreasing and takes values in a bounded and nonempty set, which also holds here.

initial distribution. The third part of the proof establishes that the stationary distribution is continuous in ω. The fourth part of the proof defines a continuous mapping of Ω on itself and applies the Schauder Fixed-Point Theorem, which, together with the first three results and the condition that Ω is a compact and convex set, implies that this mapping admits at least one fixed point. The last part shows uniqueness of the equilibrium under the specific assumptions on the workers' utility used in the paper.

Proposition 5 (Part 1) **X** *is a time-homogeneous Markov chain on a general state space.*

Equip the state space S with a boundedly compact, separable, metrizable topology $\mathscr{B}(S)$. Let $(Z, \mathscr{Z})$ be the measure space for the shocks. Let A be any subset of $\mathscr{B}(S)$. It follows that for any $x \in [V_C, \widetilde{V})$

$$P(x, A) = \begin{cases} (1-\gamma)(1-p)\alpha(V^L(x)) + \gamma & \text{if } A = \{V_0\} \text{ and } V^L(x) < V_C \\ (1-\gamma)(1-p)(1-\alpha(V^L(x)) & \text{if } A = \{V_C\} \\ (1-\gamma)(1-p) & \text{if } A = \{V^L(x)\} \text{ and } V^L(x) \geq V_C \\ (1-\gamma)p & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

And for $x = \{\widetilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma) & \text{if } A = \{\widetilde{V}\} \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

For each $A \in \mathscr{B}(S)$, $P(\cdot, A)$ is a non-negative function on $\mathscr{B}(S)$, and for each $x \in S$, $P(x, \cdot)$ is a probability measure on $\mathscr{B}(S)$. Therefore, for any initial distribution ψ, the stochastic process **X** defined on S^∞ is a time-homogeneous Markov chain. ∎

Proposition 6 (Part 2) **X** *is globally stable.*

Let **M** denote the corresponding Markov operator, and let $\mathscr{P}(S)$ denote the collection of firms distribution generated by **M** for a given initial distribution.[28]

[28]Note that Stokey, Lucas, and Prescott [1989, Theorem 12.12] fail to apply in this case because the

Write the stochastic kernel P with the density representation p so that $P(x, dy) = p(x, y)dy$ for all $x \in S$. The Dobrushin coefficient $\vartheta(p)$ of a stochastic kernel p is defined by

$$\vartheta(p) := \min \left\{ \int p(x, y) \wedge p(x', y)dy : (x, x') \in S \times S \right\} . \tag{42}$$

$(\mathscr{P}(S), \mathbf{M})$ is globally stable if $(\psi \mathbf{M}^t)_{t \geq 0} \rightarrow \psi^* \mathbf{M}$ where $\psi^* \in \mathscr{P}(S)$ is the unique fixed point of $(\mathscr{P}(S), \mathbf{M})$, which occurs if the Markov operator is a uniform contraction of modulus $1 - \vartheta(p)$ on $\mathscr{P}(S)$ whenever $\vartheta(p) > 0$. Because a firm dies with a fixed, exogenous and independent probability γ each period, and it is instantaneously replaced by a new one of size V_0, it follows that

$$P(x, \{V_0\}) \geq 0 \; \forall \; x \in S \; . \tag{43}$$

Equation (11.15) and Exercise (11.2.24) in Stachurski [2009] yield $\vartheta(p) > \gamma$. By Stachurski [2009, Th. 11.2.21], it follows that

$$||\psi \mathbf{M} - \psi' \mathbf{M}||_{TV} \leq (1 - \gamma)||\psi - \psi'|| \tag{44}$$

for every pair ψ, ψ' in $\mathscr{P}(Z)$, and where $_{TV}$ indicates the total variation norm. ∎

Proposition 7 (Part 3) *ψ^* is continuous in ω.*

The result follows if the conditions of LeVan and Stachurski [2007, Proposition 2] are satisfied.[29] Consider again the state space S equipped with a boundedly compact,

stochastic kernel is not monotone on $[V_C, a]$ where a is such that $V^L(a) = V_C$. For instance, consider the function $f(x) = x$. From the above,

$$\begin{aligned}
&\int_{[V_C,a]} P(x, dy)y \\
&= (1 - \gamma)\{(1 - p)[\alpha(V^L(x))V_0 + (1 - \alpha(V^L(x)))V_C] + pV^H(x)\} + \gamma V_0 \\
&= (1 - \gamma)[(1 - p)V_0(1 - V^L(x)/V_C) + (1 - p)V^L(x) + pV^H(x)] + \gamma V_0 \\
&= (1 - \gamma)[(1 - p)V_0\alpha(V^L(x)) + x/\beta] + \gamma V_0 \; ,
\end{aligned}$$

which is generally not increasing. The intuition is that when x falls below a, the probability of liquidation $\alpha(x)$ becomes non-zero in case of a low revenue shock. However, liquidation sends x to state V_0, which can be larger than V_C and $V^H(x)$, so that the lower the x, the higher the $\mathbf{E}(x'|x)$.

[29] LeVan and Stachurski [2007, Proposition 2] is an application of LeVan and Stachurski [2007, Theorem 1], of which Stokey et al. [1989, Theorem 12.13] is a special case.

separable, metrizable topology, $(Z, \mathscr{Z})$ a measure space, and $\mathscr{P}(Z)$ the collection of probabilities on $(Z, \mathscr{Z})$. From the above, the model can be written as

$$X_{t+1} = T_\omega(X_t, \epsilon_t), \text{ where } \epsilon_t \sim \psi_\omega \in \mathscr{P}(Z), \;\; \forall t \in \mathbb{N} \;, \tag{45}$$

where $(\epsilon_t)_{t=1}^{\infty} = (\{D_{1,t}, D_{2,t}, D_{2,t}\})_{t=1}^{\infty}$ is the vector of independently distributed binary random variables corresponding to the liquidation, revenue and death shock realization, and $T_\omega : S \times Z \to S$ is measurable. Given the price vector ω, the stochastic kernel can be written as $P_\omega(x, B) := \psi_\omega\{z \in Z : T_\omega(x, z) \in B\}$, and given the parameter space Ω, the family of stochastic kernel is $\{P_\omega : \omega \in \Omega\}$. Let N be any subspace of Ω, and define $\Lambda(\omega) := \{\mu \in \mathscr{P}(S) : \mu = \mu P_\omega\}$ the collection of invariant distribution corresponding indexed by ω.

Lemma 1 (LeVan and Stachurski [2007]) *If $\Lambda(\omega) = \{\mu_\omega\}$, then $\omega \mapsto \mu_\omega$ is continuous on N if the following four conditions are satisfied:*

1. *the map $N \ni \omega \mapsto T_\omega(x, z) \in S$ is continuous for each pair $(x, z) \in S \times Z$*
2. *for each compact $C \subset S$, there is a $K < \infty$ with*

$$\int d(T_\omega(x, z), T_\omega(x', z))\psi_\omega(dz) \leq Kd(x, x'), \forall x, x' \in C, \forall \omega \in N \tag{46}$$

3. *$\exists$ a Lyapunov function $\mathcal{V} \in \mathscr{L}(S)$, $\lambda \in (0, 1)$, and $L \in [0, \infty)$ s.t. $\forall \omega \in N$*

$$P_\omega \mathcal{V}(x) := \int \mathcal{V}(T_\omega(x, z))\psi_\omega(dz) \leq \lambda \mathcal{V}(x) + L \quad \forall x \in S \tag{47}$$

4. *$\omega \mapsto \psi_\omega$ is continuous in total variation norm*

That $\Lambda(\omega)$ is nonempty, and $\Lambda(\omega) = \{\mu_\omega\}$ for each $\omega \in N$ follows from Proposition 1. Condition (1) requires the optimal value function $W(x)$ to be continuous in ω, which follows from Berge's theorem (see, for example, Ausubel and Deneckere [1993]). Condition (4) holds as the shocks are independent and the probability of liquidation is $\alpha = (x - V_C)/V_C$, which is continuous in ω because V_C is continuous in ω from condition

(1). To show that condition (2) holds, define again $a \ni V^{LL}(a) = V_C$ and $b \ni V^{HH}(b) \geq \widetilde{V}$. Pick any $x, x' \in C \subset [V_C, a)$. Without loss of generality, assume $x > x'$ so that $\alpha(V^L(x')) > \alpha(V^L(x))$. By noting that $\alpha(V^L(x')) - \alpha(V^L(x)) = (V^L(x') - V^L(x))/V_C$, and $x = \beta[pV^H(x) + (1-p)V^L(x)]$ at optimum, it follows that

$$\begin{aligned} & \int d(T_\omega(x,z), T_\omega(x',z))\psi_\omega(dz) \\ < \; & |pV^H(x) + (1-p)V^L(x) - pV^H(x') - (1-p)V^L(x')| \\ = \; & \frac{1}{\beta}|x - x'| = \frac{1}{\beta}d(x,x'). \end{aligned}$$

The above inequality also holds for any $x, x' \in C \subset [a, b)$. Last, recall that $V^L(x) = (x - pF(R(x)))/\beta$. So, for any $x, x' \in C \subset [b, \widetilde{V})$

$$\begin{aligned} & \int d(T_\omega(x,z), T_\omega(x',z))\psi_\omega(dz) \\ < \; & (1-p)|V^L(x) - V^L(x')| \\ = \; & \frac{(1-p)}{\beta}|x - x'| = \frac{(1-p)}{\beta}d(x,x'). \end{aligned}$$

It remains to show that condition (3) holds. Pick $\mathcal{V}(x) = x$, which is a Lyapunov function, as S is boundedly compact. Then,

$$P_\omega x = \begin{cases} (1-\gamma)\{(1-p)[\alpha(V^L(x))V_0 + (1-\alpha(V^L(x)))V_C] + pV^H(x)\}\mathbf{1}_{[V_C,a)}(x) \\ + [pV^H(x) + (1-p)V^L(x)]\mathbf{1}_{[a,b)}(x) \\ + \widetilde{V}\mathbf{1}_{(x=\widetilde{V})}(x)\} + \gamma V_0 \end{cases} . \tag{48}$$

Pick any $x \in [V_C, a)$ so that $V_C \leq x \leq V^H(x)$. Then,

$$\begin{aligned} P_\omega x \; & < \; (1-p)(1-\alpha(V^L(x)))V_C + pV^H(x) + [(1-p)\alpha(V^L(x)) + \gamma]V_0 \\ & \leq \; (1-p)V_C + pV^H(x) + V_0 \\ & \leq \; \lambda x + \sup_{\omega \in N} V_0 = \lambda\mathcal{V}(x) + L \end{aligned}$$

The same inequality holds for any $x \in [a, b)$, as $V^L(x) < x < V^H(x)$. Last, when

$x = \widetilde{V}$,

$$\begin{aligned} P_\omega x &= (1-\gamma)\widetilde{V} + \gamma V_0 \\ &\leq \lambda x + \sup_{\omega \in N} V_0 \ . \end{aligned}$$

■

Proposition 8 (Part 4) *There exists an equilibrium.*

Using the capital and labor market clearing conditions, define the mapping

$$\mathbf{f}(\omega) = \begin{bmatrix} D_w(\omega) + D_e(\omega) - \int R(V,\omega)d\mu(\omega) - \hat{\gamma}(\omega)(I_0 - S) - \gamma I_0 \\ \int n(V,\omega)d\mu(\omega) - H \end{bmatrix} .$$

such that $\mathbf{f} : \Omega \mapsto \mathbb{R}^2$. Prices r and w must each be positive and greater than zero. Without loss of generality, assume that r and w are bounded above by arbitrarily large but finite numbers $\bar{r}$ and $\bar{w}$. It follows that the set Ω is compact and convex. Define the mapping $\Phi : \Omega \mapsto \Omega$ such that

$$\Phi(\omega) = \operatorname*{argmax}_{\omega \in \Omega} -||\mathbf{f}(\omega)||^2 \tag{49}$$

From Proposition 6 and Proposition 8, the maximand is continuous in ω so that the correspondence Φ is also continuous. Applying the Schauder Fixed-Point [Stokey et al., 1989, Theorem 17.4] yields the results. ■

Proposition 9 (Part 5) *The equilibrium is unique.*

Uniqueness is difficult to establish with more general preferences than the one used in the text. When the agents' instantaneous utility is linear in consumption, the capital market trivially clears for a given wage rate, which is the only relevant price. Under this assumption there is no wealth effect, and given a value for r, an increase in w yields a lower aggregate demand for labor $\int n(R(V))d\mu$ and a higher aggregate supply of labor, defined implicitly by $\varphi'(H) = w$. It follows that there exists a unique w that clears the labor market. ■

A.2 Clearing of the goods market

The definition of aggregate output and entrepreneurs' consumption implies that

$$Y = p \int f(R(V))d\mu \tag{50}$$

$$= C_e + p \int \tau(V)d\mu \ , \tag{51}$$

where C_e is the aggregate consumption of entrepreneurs. The representative financial intermediary budget must be balanced each period so that

$$D_e' = (1+r)D_e + p\int \tau(V)d\mu - (1+r)\int R(V)d\mu + (1-\delta)\int kd\mu - \Gamma K_0 \ ,$$

where $\Gamma K_0 = \gamma I_0 + \hat{\gamma}(I_0 - S)$ is the net aggregate fixed investment in new firms. In the steady state, the balance budget condition implies that

$$-rD_e = p\int \tau(V)d\mu - (1+r)\int R(V)d\mu + (1-\delta)\int kd\mu - \Gamma K_0 \ .$$

Multiplying the capital market-clearing condition by r yields

$$rD_e + rD_w = r\int R(V)d\mu + r\Gamma K_0 \ .$$

Note that a negative D_e means that the financial intermediary borrows form the representative worker to finance the firms. Substituting for rD_e in the intermediary balance budget condition and rearranging the terms yields

$$p\int \tau(V)d\mu = rD_w - r\Gamma I_0 + \int R(V)d\mu - (1-\delta)\int kd\mu + \Gamma K_0$$

It follows that

$$Y = C_e + rD_w + \int R(V)d\mu - (1-\delta)\int k(V)d\mu + \Gamma K_0 \tag{52}$$

$$= C_e + rD_w + w\int n(V)d\mu + \delta\int k(V)d\mu + \Gamma K_0 \ . \tag{53}$$

Finally, using the labor market-clearing condition and the aggregate budget constraint for workers,

$$C_w + D_w = wH + (1+r)D_w \ , \tag{54}$$

yields

$$Y = C_e + C_w + I \ , \tag{55}$$

where $I = \delta \int k(V) d\mu + \Gamma K_0$.

A.3 Other proofs

This section sketches the remaining proofs of propositions in the text. To economize on notation, I use Greek letters for Lagrange multipliers that conflict with some of the notation used in the main text but should not be a source of significant confusion in this section.

A.3.1 Proof of Proposition 3

When $m \to \infty$ and $\phi(e)$ is low enough, the choice of e enters the contracting problem such that, abstracting from the liquidation decision, the optimal contract maximizes the value of the joint surplus:

$$W(V) = \max_{R, V^L, V^H, e} pf(R) + (1-\delta)k(R) - (1+r)(R + \phi(e)) + \beta(pW(V^H) + (1-p)W(V^L)) \tag{56}$$

$$\text{s.t. } \lambda : V \geq \beta(pV^H + (1-p)\beta V^L) \tag{57}$$

$$\mu : \beta V^H \geq f(R) + \beta V^L \tag{58}$$

$$\epsilon : \beta V^L \geq \mathcal{O} - e \tag{59}$$

given that $\tau = f(R)$ and limited liability is satisfied in equilibrium. The Lagrange multipliers associated with each of the constraints are indicated on the left-hand side.

The first order conditions and the Envelope condition for this problem are:

$$R: \quad pf'(R) + (1-\delta)k'(R) - (1+r) - \mu f'(R) = 0 \tag{60}$$

$$V^H: \quad pW'(V^H) - p\lambda + \mu + \beta = 0 \tag{61}$$

$$V^L: \quad (1-p)W'(V^L) - (1-p)\lambda - \mu + \epsilon = 0 \tag{62}$$

$$e: \quad -(1+r)\phi'(e) + \epsilon = 0 \tag{63}$$

$$\text{EC}: \quad W'(V) - \lambda = 0 \tag{64}$$

Rearranging the first-order conditions and the Envelope condition yield

$$f'(R)\left[\phi'(e)(1+r) + (1-p)\left(W'(V^L) - W'(V)\right)\right] = pf'(R) + (1-\delta)k'(R) - (1+r)\ . \tag{65}$$

When the enforcement constraint conditional on a low revenue shock does not binds, $\epsilon = 0$ and $-(1+r)\phi'(e) = 0$. Because $\phi(\cdot)$ is strictly convex, $e(V) = 0$ in this case and whenever $V^L > \mathcal{O}$. When this constraint binds and $\epsilon > 0$, $\beta V^L = \mathcal{O} - e$, which implies that

$$f'(R)\left[\phi'(e)(1+r) + (1-p)\left(W'\left(\frac{\mathcal{O}-e}{\beta}\right) - W'(V)\right)\right] = pf'(R)+(1-\delta)k'(R)-(1+r)\ . \tag{66}$$

In this case, an increase in e for a given V allows for a higher R because it increases $W'\left(\frac{\mathcal{O}-e}{\beta}\right)$ and $W'\left(\frac{\mathcal{O}-e}{\beta}\right) > W'(V)$ because W is strictly concave. Moreover, the first order condition and the Envelope condition yield

$$(1+r)\phi'(e) = W'(V) - \left((1-p)W'(V^L) + pW'(V^H)\right)\ .$$

Because $V^L < V < V^H$, $W(V)$ is strictly concave and $\phi(e)$ is strictly convex, the promise-keeping constraint $V/\beta = ((1-p)V^L + pV^H)$ and Jensen's inequality implies that

$$(1+r)\phi'(e) > W'(V) - W'(V/\beta)\ ,$$

as $\beta < 1$. Given that $\phi(\cdot)$ is strictly convex and $W(V)$ is concave, it follows that higher V are generally associated with a lower e to maintain optimality, but $e(V)$ is not necessarily monotonic in V .

A.3.2 Proof of Proposition 4

When $m = 0$ and $\phi(e)$ is low enough and again abstracting from the liquidation decision, the optimal contract maximizes the value of the joint surplus:

$$W(V) = \max_{R,V^L,V^H,e} pf(R) + (1-\delta)k(R) - (1+r)(R + \phi(e)) + \beta(pW(V^H) + (1-p)W(V^L)) \tag{67}$$

$$\text{s.t. } \lambda : V \geq \beta(pV^H + (1-p)\beta V^L) \tag{68}$$

$$\mu : \beta V^H \geq f(R) + \mathcal{O} - e \tag{69}$$

$$\epsilon : \beta V^L \geq \mathcal{O} - e \tag{70}$$

given that $\tau = f(R)$ and limited liability is satisfied in equilibrium. Thus, the only difference between this maximization problem and the one considered previously is the incentive constraint, which no longer needs to be satisfied. The first-order conditions and the Envelope condition for this problem are

$$R: \quad pf'(R) + (1-\delta)k'(R) - (1+r) - \mu f'(R) = 0 \tag{71}$$

$$V^H: \quad pW'(V^H) - p\lambda + \mu = 0 \tag{72}$$

$$V^L: \quad (1-p)W'(V^L) - (1-p)\lambda + \epsilon = 0 \tag{73}$$

$$e: \quad -(1+r)\phi'(e) + \mu + \epsilon = 0 \tag{74}$$

$$\text{EC}: \quad W'(V) - \lambda = 0 \; . \tag{75}$$

Rearranging yields the same expression as before:

$$f'(R)\left[\phi'(e)(1+r) + (1-p)\left(W'(V^L) - W'(V)\right)\right] = pf'(R) + (1-\delta)k'(R) - (1+r) \; ,$$

recalling that the intermediary terminates the contract with zero transfer if the entrepreneur is found misreporting and promises V^L to an entrepreneur truthfully reporting a low cash flow shock. Note that the enforcement constraint conditional on a low realization is never binding so that $\epsilon = 0$. To see this effect, assume for the purpose of contradiction that it binds and $\epsilon > 0$ so that $\beta V^L = \mathcal{O} - e$. Substitution into the promise keeping constraint yields $V \geq \beta(pV^H + (1-p)\beta(\mathcal{O} - e))$. However, it must also be that $V \geq \mathcal{O} - e$, which violates the enforcement constraint conditional on a high cash flow shock. Given $\epsilon = 0$, $W'(V^L) = \lambda$ and the Envelope condition implies that $V^L(V) = V$. In this case, promise-keeping implies that $V^H(V) = V(1 - \beta(1-p))/(\beta p)$ and the equilibrium condition above simplifies to

$$f'(R)\phi'(e)(1+r) = pf'(R) + (1-\delta)k'(R) - (1+r) \ .$$

It follows that $e = 0$ when $R = \tilde{R}$, as $pf'(\tilde{R}) + (1-\delta)k'(\tilde{R}) = (1+r)$ which happens for all $V \geq V_u$. Similarly, $e > 0$ when $R < \tilde{R}$ and an increase in e is associated with an increase in R when $V < V_u$. As before, the first order condition and the Envelope condition yield

$$(1+r)\phi'(e) = W'\left(V\right) - \left((1-p)W'\left(V^L\right) + pW'\left(V^H\right)\right) \ ,$$

which simplifies to

$$(1+r)\phi'(e) = pW'(V) + pW'\left(V(1-\beta(1-p))/(\beta p)\right)$$

since $V^L(V) = V$ when monitoring is free. This last equality implies that e is strictly decreasing in V, as $(1-\beta(1-p))/(\beta p) > 1$ with $\beta < 1$

A.4 Parameter estimation and numerical solution

This section provides an overview of the algorithms used to estimate some of the model parameters using a simulated method of moment (SMM) procedure with firm-level data from Colombia in the 1980s and early 1990s and solve for the steady state and the transition dynamics of the benchmark economy.

A.4.1 A useful first step

A useful first step is to normalize some of the parameters to make the optimal contract invariant to the wage rate w.

Proposition 10 *Given z is chosen to normalize $\tilde{R}$ to 1, the optimal contract is invariant to the wage rate w if I_0 is expressed as a fraction $x \in (0,1)$ of the maximum joint surplus $W(\tilde{V})$ and S as a fraction of I_0.*

This result follows from the observation that

$$V_0 = \sup_V \{W(V) - V = I_0\} = \sup_V \{W(V) - V = x \cdot W(\tilde{V})\} \ ,$$

and because $\tilde{V} = \frac{pf(1)}{1-\beta}$ is invariant to w, so is V_0 and the optimal value function $W(V)$

$\blacksquare$

This simple observation considerably speeds up the algorithms used to solve for the general equilibrium in the eight versions of the model economy as the optimal contract in each of these economy only needs to be computed once. This result also permits the structural estimation of the parameters of the benchmark contract using data on a subset of firms in the economy without having to explicitly consider (or ignore) general equilibrium effects.

A.4.2 Estimating the parameters of the contract

The data used to estimate the contract parameters is a subset of the database constructed by Eslava et al. [2004] from the Colombian Annual Manufacturing Survey

(AMS) for 1982 to 1998.[30] I focus on the period extending from 1982 to 1992, which predates a number of wide ranging trade and financial reforms in Colombia. For each plant in each of the 29 three-digit manufacturing sectors and in each year, I observe its capital stock for buildings and structures, capital stock for machinery and equipment, and total factor productivity (TFP). The two capital variables are computed using perpetual inventory methods, and physical quantities are calculated by deflating nominal values by the appropriate plant-level material price index. The measure of plant-level TFP is estimated by the authors using plant-level physical output and elasticities estimated using downstream industry demand to instrument input. I add to this data observed industry-level depreciation rates for buildings and structures and for machinery and equipment reported by Pombo [1999]. These depreciation rates are the one used by Eslava et al. [2004] to compute the two type of plant-level capital stock. A firm identifier allows for tracking firms throughout the sample period. A plant is classified as entering in year t if it exists in t but not $t-1$. Similarly, a plant exits if it is present in t but not in $t+1$. Consequently, exit in the data means either true exit or exit to the micro establishment level with less than 10 employees. Lastly, although this data is at the plant level, the vast majority of manufacturing firms in Colombia at that time were single-plant firms.

I estimate the depreciation rate δ and the parameter p governing the firm cash flow process directly in the panel. I use the two capital stock variables and the corresponding industry-level depreciation rate of Pombo [1999] to back out the corresponding measure of plant-level investment in building/structures and in equipment/machinery. Summing over the two types of capital yields a measure k_{ijt} and i_{ijt} of plant-level capital stock and investment of plant i in industry j in year t. After windsorizing k_{ijt} and i_{ijt} at the 1 percent level in each industry to reduce the influence of outliers, I estimate the industry-level average depreciation rates by regressing $(-k_{ijt} + k_{ijt-1} + i_{ijt})$ on k_{ijt-1}

[30] The AMS is housed at the Colombian Departamento Administrativo Nacional de Estadística (DANE). The database constructed by Eslava et al. [2004] using information from the AMS is archived by the Dirección de Metodología y Producción Estadística at DANE and can be accessed through DANE for approved projects and for statistical purposes only. See the online documentation for Eslava et al. [2013] posted on the Review of Economic Dynamics website for more information, which is available at https://economicdynamics.org/codes/11/11-69/readme.txt.

interacted with industry dummies and without a constant term. The average of these 29 industry-level depreciation rates (the coefficient estimates of regression) is an estimate of the average Colombian manufacturing sector that controls for industry fixed effects. I then calculate the standard deviation of the average depreciation rate with respect to the industry-level estimates.

In the model, the shocks to firm cash can also be interpreted as shocks to firm TFP with which firm productivity is 1 with probability p and 0 with probability $(1-p)$. Using Eslava et al. [2004]'s estimate of plant-level TFP and assuming normality of the distribution of $\log TFP$, I approximate the distribution of TFP in industry j with a two-point distribution with parameter p_j. Averaging p_j across industries yields an estimate of p that controls for industry fixed effects. As for the estimate of the sector depreciation rate, I calculate the standard deviation of the p estimate with respect to the industry-level estimates.

It remains to assign values to the firm setup cost I_0, salvage value S, and the exogenous exit rate γ. The parameters I_0 and S play an important role in shaping firm and industry dynamics in the model but do not have a clear counterpart in the data. In general, $I_0 - S$ is the portion of the initial investment that is sunk. A higher S is typically associated with higher initial value V_0 and greater exit due to liquidation. Clementi and Hopenhayn [2006] show that a higher S also implies that $R(V)$ is higher for all $V < \tilde{V}$, which is also true in the benchmark contract. For a given S, a higher I_0 reduces the maximum level of initial debt an intermediary can optimally commit to, which reduces the starting value V_0 and tightens the credit constraint of young firms. As a result, the investment of small firms tends to be more volatile. Taken together, the optimal contract implies that the standard deviation of small firms' investment and the exit rate of small firm are increasing in I_0 and S, respectively. Moreover, the volatility of small firms' investment and the exit rate of small firms tend to decrease with a higher S and I_0, respectively. These monotonic relationships between model-generated moments and model parameters are the basis to identify I_0, S and γ using a SMM procedure using the plant-level data. The SMM procedure consists of finding values for I_0, S and γ that make a vector of moments generated by the model as close as possible to their analog in the actual data. The SMM procedure targets four moments: the investment volatility of small firms, the exit rate of small firms, the exit rate of firms that are not small and the overall firm exit rate.

A first issue is to decide what constitutes a small firm. I choose the standard deviation of investment for firms in the bottom 25 percent of size distribution, where size is measured by firm capital stock, and the exit rate of firms above and below the 10 percent threshold of the firm distribution. The estimation procedure is not really sensitive to the 10 percent size threshold for the exit rate, as a smaller size threshold mechanically implies the exit rate of firms below is higher. What is important for identification is that given a particular threshold (in this case 10 percent), the exit rate of firms below the threshold is higher than the exit rate of firms (plants) above the threshold. The 25 percent size threshold for the firm investment volatility is more arbitrary and largely guided by sample size consideration—i.e., one-quarter of the plants in each industry. I found that the plant investment volatility moment estimate in the data is not substantially different around this threshold.

Another issue is that firms in the model are ex-ante identical and subject to i.i.d. idiosyncratic shocks. As explained above, the contract can be made invariant to prices with a suitable normalization of the parameters. Given this normalization, bringing the model to data requires either adding firm and/or industry heterogeneity to the model or removing this heterogeneity from the data. I follow Hennessy and Whited [2005] and DeAngelo et al. [2011] and choose the latter, which considerably reduces the computational burden associated with the SSM procedure. To do so, I first estimate the exit rate of plants in the data that are below and above the 10 percent size threshold in their respective industry using a linear regression with industry and year fixed effects. I then estimate the standard deviation of firm-level investment by computing the standard deviation of the residuals from a regression of firm investment on firm and year fixed effects.

As explained by Hennessy and Whited [2005], an optimal weighting matrix for SMM is the inverse of the covariance matrix of the actual data moments and only needs to be computed once.[31] To proceed with the estimation of the data moment covariance matrix, I implement a "block bootstrap" procedure that amounts to sampling firm life cycles with replacement in the actual data set to create a collection of bootstrapped data sets with the same number of firms.[32] Sampling entire firm lifecycles with replacement is essential to preserve the panel characteristics—i.e., serial correlation and heteroschedasticity—of the original data set.[33] I follow Cameron et al. [2008] and use 9,999 bootstrap replications to estimate the data moment covariance matrix.

The rest of the SMM procedure is standard and consists of finding parameters that makes the simulated moments as close as possible to the actual data moments and then using a numerical approximation of the first-order derivatives of the parameter vector to compute their standard errors. Following the discussion of finite-sample performance in DeAngelo et al. [2011], I simulate the model 10 times at each iteration of the SMM optimization procedure (with the same seeds) to obtain 10 artificial panels with the same number of firms as in the actual data set.

A.4.3 Solving for the steady state

(1) Guess a value for w from a compact set.

(2) Guess a value for $\bar{V}$ from a compact set.

(3) Solve the contract using value iteration on a grid. It is quite essential to use shape-preserving Splines as the contract requires solving for non-linear equations involving derivatives of the iterated value function.

[31] Hennessy and Whited [2005] recommend using influence functions to estimate the moments' covariance matrix. This method is not applicable in this case because the three data moments of interest are computed on three different and mutually exclusive partitions of the data. For example, the exit rate of small firm is estimated using data on firms that are below the 10 percent size threshold, and the exit rate of large firms is estimated using the complement of this data set.

[32] Consequently, the bootstrapped data sets will not have exactly the same number of observations as the actual data set, though the number of observations will be close.

[33] This "block bootstrap" procedure is inspired by the "cluster bootstrap" procedure of Cameron, Gelbach, and Miller [2008] used to correct standard error in certain regression models when the number of clusters is small. As far as I am aware, the "block bootstrap" procedure discussed here has not been used in the context of SMM before and potentially provides a useful alternative to other procedures that are not applicable when the data moments are computed on different partitions of the actual data.

(4) Solve for the initial value from $V_0 = \sup_V\{W(V) - V = (1+r)I_0\}$. If $V_0 \neq \bar{V}$ go to (2), if $V_0 = \bar{V}$ go to (5).

(5) Estimate the invariant distribution using the Look-Ahead Estimator as described in Stachurski and Martin [2008].

 (1') Check labor market clearing: if it does not clear, guess a new w using bisection and go to (2'); if it clears go to (1").

 (2') Guess a new value for $\bar{V}$ using bisection.

 (3') Solve the contract using value iteration on a (non-linear) grid, and solve for the initial value from $V_0 = \sup_V\{W(V) - V = I_0\}$. If $V_0 \neq \bar{V}$, go to (2'), if $V_0 = \bar{V}$ go to (4'). Note that the program is now using the old distribution as an approximation because it is not too different around a particular value of w, which increases the efficiency of the algorithm substantially.

 (4') Go to (1').

(6) Stop when the maximum absolute difference between supply and demand in the labor market falls below the desired tolerance level.

A.4.4 Computing transition dynamics

Section 6 investigates the transition dynamics of the benchmark economy in the years following a reform that eliminates limited enforcement and private information frictions. In these exercises, a reform is unexpected and permanent. The reform is implemented at the end period t_0 and after the promised continuation values are awarded to entrepreneurs. At the beginning of the next period, $t_0 + 1$, the optimal contract takes as given the sequence of equilibrium prices $\{w_\iota\}_{\iota=t_0+1}^{\infty}$. Let $\mathcal{H}$ denote the law of motion for the distribution of promise continuation values μ_t, such that $\mu(\{w_\iota\}_{\iota=t+1}^{\infty}) \sim \mathcal{H}(\mu(\{w_\iota\}_{\iota=t}^{\infty}))$. The definition of an equilibrium in the transition is given as follows

Definition 2 *Given a sequence of prices $\{w_\iota\}_{\iota=t_0+1}^{\infty}$, an interest rate r, and an initial distribution μ_{t_0}, an equilibrium consists of labor supply and consumption function h_t*

and c_t for workers, a contract $\{R_t, \tau_t, V_t^L, V_t^H, \alpha_t, Q_t, V_{Ct}\}$, an initial firm value $V_{0,t}$, , a mapping T, and a law of motion $\mathcal{H}$, such that in every period $t > t_0$:

1. *the labor and consumption functions maximize workers' utility*
2. *the financial contract maximizes the value of the firm*
3. *$V_{0,t}$ is such that the intermediary breaks even on new contracts*
4. *$\mathcal{O}_t$ is the fixed point of T*
5. *individual decisions are consistent with $\mathcal{H}$*
6. *the labor and capital market clear*

This algorithm is a brute force method to iteratively solve for the exact sequence of prices and state-contingent optimal decision rules such that all markets in every period. Although this algorithm converges and allows for an explicit aggregation of the decisions of a very large number of agents, it is quite computationally intensive.

(1) Simulate the economy with a large number of firms (I use $40,000 \times 25$ to optimize the load across processors on the server) for a large number of period T until the economy reaches its steady state, and use it as the $t = 0$ firm distribution. Although the prices computed using the Look-Ahead Estimator are asymptotically equal to the one computed using the marginal distribution of firms, they will differ slightly in a finite sample. Consequently, this step requires iteratively solving the steady prices using the marginal distribution once.

(2) Generate the sequences of idiosyncratic shocks.

(3) At $t = 1$, implement the reform and guess a sequence of prices.

  (1') Guess a value for $\bar{V}$ from an interval.

  (2') Solve the contract using value iteration on a grid, and get the initial value from $V_0 = \sup_V \{W(V) - V = (1 + r)I_0\}$. If $V_0 \neq \bar{V}$ go to (1'), if $V_0 = \bar{V}$ go to (3').

(3') Update the distribution of firms given the realization of idiosyncratic shocks.

(4') Estimate the marginal distribution of firms (Stachurski and Martin [2008]), and check the market clearing conditions. If markets do not clear in all periods, find the prices that would have cleared the markets in each period using the estimate of the firm distribution. Finding these prices requires solving the system of non-linear equations from the market-clearing conditions using the estimate of the firm distribution to integrate.

(5') Update the sequence of prices and go to (2).

(4) Iterate until the maximum discrepancy from the market-clearing conditions falls below the desired threshold and the sequence of prices no longer changes.

A.5 Additional results on the general equilibrium effects along the transition path

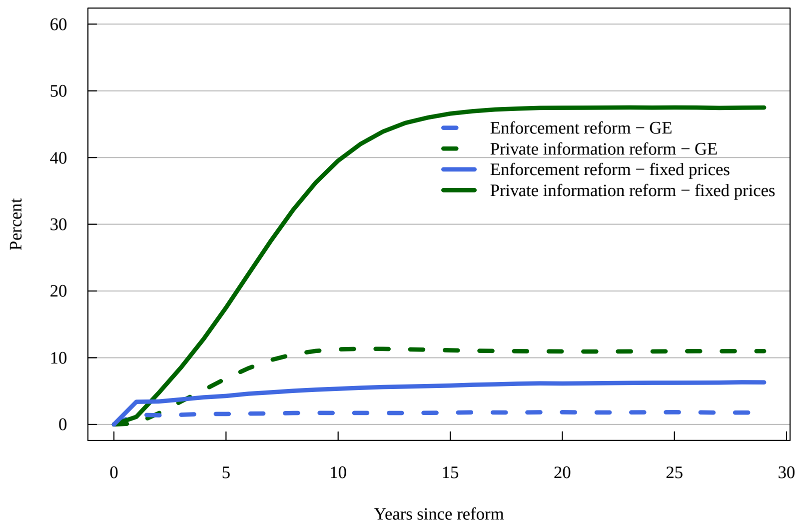


Figure 5: Transition dynamics with fixed prices and in general equilibrium

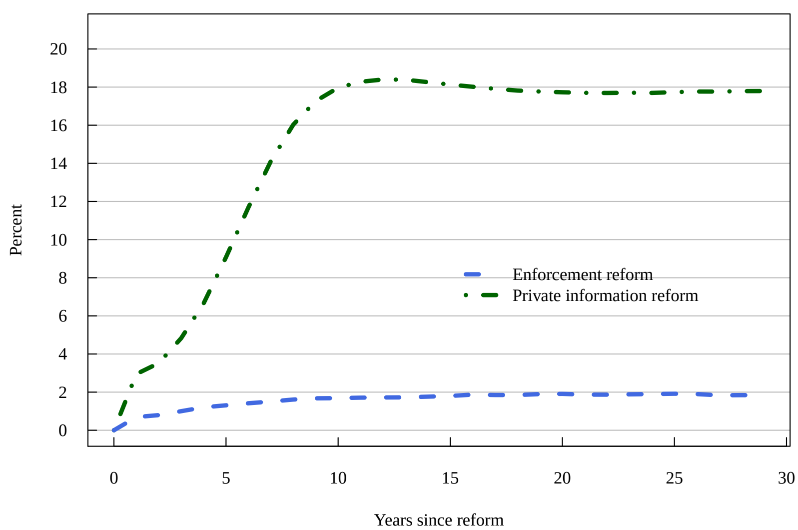


Figure 6: Aggregate capital stock.

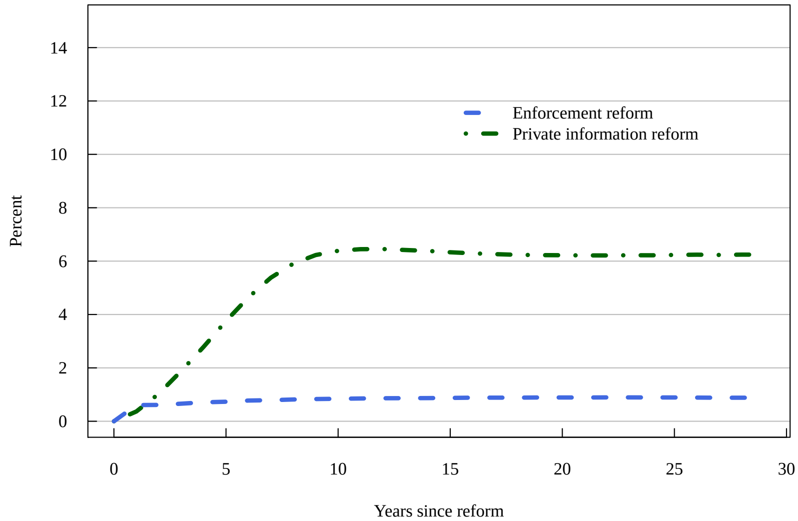


Figure 7: Wages.

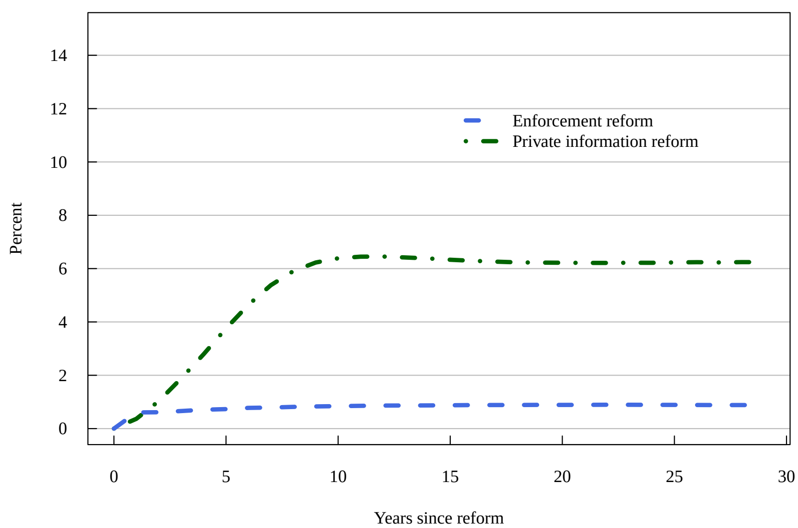


Figure 8: Hours.